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Optimisation of the Competency Assessment System Through Matrix Applications and Linear Algebra Using the AHP Method

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Abstract - Competency-based assessment systems are increasingly important in education and industry to objectively assess individual abilities, overcoming the subjectivity issues inherent in traditional assessment methods. This study aims to develop an innovative competency assessment system by combining Assessment Matrix and Linear Algebra, specifically using the Analytic Hierarchy Process (AHP) method to systematically and accurately determine the weight of criteria. The research data were taken from a dataset of college students, with five main criteria of competence, including technical skills, cooperation, and creativity. The data normalization process was carried out using Min-Max Scaling and Z-Score Normalization to ensure consistency, followed by the construction of an AHP comparison matrix based on the level of importance between criteria. The weight of the criteria was calculated using the eigenvector method, and the consistency test was carried out through the Consistency Ratio (CR) to ensure the validity of the matrix (CR < 0.1). The final assessment was obtained by multiplying the AHP weights by the student's scores for each criterion. The results showed that this approach resulted in a more objective, transparent, and accurate assessment system than conventional methods, with the potential to improve fairness in evaluation in the academic environment. This research provides a new contribution in the application of linear algebra to the development of competency assessment systems, as well as offering practical solutions for educators and human resource managers in improving performance evaluation.

Keywords :
Competency Based; Assessment System; Assessment Matrix; Linear Algebra; Data Normalization;

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1. INTRODUCTION

Competency-based assessment systems have become the primary method in academic and professional settings for assessing individuals' abilities more accurately and objectively. In education, competency assessment is used to evaluate students' understanding of the material taught (Santoso et al., 2023). On the other hand, in the context of work, this system serves to assess employees' skills and performance holistically (Setiawan, 2021). This approach has advantages over traditional assessment methods, which are often subjective and do not consider the multidimensional aspects of competency (Rahayu, 2020).

Although competency-based assessment can provide more reliable evaluations, traditional methods often do not systematically consider the relative importance of criteria, which can lead to inconsistent and inaccurate assessment results (Wijayanti, 2022). For example, research by Rahayu (2020) revealed that 65% of educational institutions still use conventional assessment systems that rely solely on average scores without considering variations in competency criteria. Therefore, a more structured and objective approach to assessment is needed.

To address this issue, assessment matrix and linear algebra-based methods can be applied in competency evaluation. One popular technique in multi-criteria decision making is the Analytic Hierarchy Process (AHP), which allows for more objective assessment by taking into account the relative importance of each criterion (Santoso et al., 2023). Setiawan's (2021) research shows that integrating AHP with data normalisation increases the accuracy of student competency assessment by up to 30% compared to conventional methods. Using AHP, the weight of each criterion can be calculated mathematically using the eigenvector method, producing objective values for each criterion.

In addition, it is important to perform a consistency test (Consistency Ratio/CR) on the comparison matrix used in AHP. The study by Santoso et al. (2023) emphasises that CR < 0.1 is a key indicator to ensure that there is no excessive bias and that the decision-making process is reliable. This is in line with the findings of Wijayanti (2022), who states that inconsistencies in the AHP matrix can cause criterion weight deviations of up to 22%.

Data normalisation is also crucial in assessment systems, especially if the dataset has different value scales. Techniques such as Min-Max Scaling and Z-Score Normalisation have proven effective in equalising value scales so that subsequent analyses are more fair and accurate (Wijayanti, 2022). Setiawan's (2021) research also confirms that combining AHP with Z-Score normalisation produces more stable assessments that are resistant to outliers. By integrating these various methods, competency assessments can become more objective, structured, and transparent, resulting in more accurate and accountable evaluations.

2. RESEARCH METHOD

This study developed a competency-based assessment system utilising an assessment matrix and linear algebra through a web application built using the Streamlit framework. The system was designed to improve the objectivity of student assessment by integrating the Analytic Hierarchy Process (AHP) method and data normalisation techniques. The system was implemented through the following steps:

System Workflow
Data Input: Users upload Excel files containing student grade datasets via the Streamlit interface.
Criteria Column Selection: The system automatically selects 5 main columns as assessment criteria:

- **PRE** (Pre-test score)
- **TGS** (Task value)
- **TB 1** (Grade for major assignment 1)
- **TB 2** (Grade for major assignment 2)
- **UAS** (Final exam scores)

If the column is not available, the system takes the first 5 columns as default.

Handling of Lost Data: Missing values are filled with 0 to ensure data completeness.

Data Normalisation Method

Objective: Addressing differences in scale between criteria (e.g. scores of 0–100 vs 0–50) to enable fair comparison.

Before performing further calculations, student scores need to be normalised so that they have a uniform scale. The two methods used are:

a. **Min-Max Scaling**, used to convert values into the range [0,1] with the formula:

$$X^{-1}= \frac{x-x_{min}}{x_{max}-x_{min}}$$

b. **Z-Score Normalization,** used to standardise values based on data distribution using a formula:

$$Z= \frac{x-\mu}{\sigma}$$

where μ is the mean and σ is the standard deviation.

Weighting Criteria with AHP

Criteria weighting was carried out using the Analytic Hierarchy Process (AHP) method, which consisted of the following steps:

a. **a. Compiling a criteria comparison matrix** based on the relative importance of each criterion.

Table 1. Pairwise Comparison Matrix

Skala Saaty (1–9):

Skala	Arti	Example
1	Both criteria are equally important.	PRE vs TGS are considered equivalent
3	Criterion A is slightly more important	TB1 is slightly more important than PRE
5	Criterion A is more important	The final exam is more important than TB1
9	Criterion A is more important.	The final exam is much more critical than the midterm exam

Example of an AHP Matrix (for 5 criteria):

$$A = \begin{bmatrix} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\ 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\ 2 & 2 & 1 & 1 & \frac{1}{2} \\ 2 & 2 & 1 & 1 & \frac{1}{2} \\ 3 & 3 & 2 & 2 & 1 \end{bmatrix}$$

Figure 1. AHP Matrix

Matrix Normalisation:

Each column is divided by the total number of columns.

Calculate the total for each column:

- Column 1: 1+1+2+2+3=91+1+2+2+3=9
- Column 2: 1+1+2+2+3=91+1+2+2+3=9
- Column 3: 1/2+1/2+1+1+2=521+21 +1+1+2=5
- Column 4: 1/2+1/2+1+1+2=521+21 +1+1+2=5
- Column 5: 1/3+1/3+1/2+1/2+1=2.6667

Normalised Matrix (Anorm):

$$A_{\text{norm}} = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{0.5}{5} & \frac{0.5}{5} & \frac{0.333}{2.6667} \\ \frac{1}{9} & \frac{1}{9} & \frac{0.5}{5} & \frac{0.5}{5} & \frac{0.333}{2.6667} \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{5} & \frac{1}{5} & \frac{0.5}{2.6667} \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{5} & \frac{1}{5} & \frac{0.5}{2.6667} \\ \frac{3}{9} & \frac{3}{9} & \frac{2}{5} & \frac{2}{5} & \frac{1}{2.6667} \end{bmatrix}$$

Figure 2. Normalised matrix Anorm

$$A_{\text{norm}} = \begin{bmatrix} 0.111 & 0.111 & 0.100 & 0.100 & 0.125 \\ 0.111 & 0.111 & 0.100 & 0.100 & 0.125 \\ 0.222 & 0.222 & 0.200 & 0.200 & 0.188 \\ 0.222 & 0.222 & 0.200 & 0.200 & 0.188 \\ 0.333 & 0.333 & 0.400 & 0.400 & 0.375 \end{bmatrix}$$

Figure 3. Anorm Normalisation Calculation Results

b. Calculate the eigenvector to determine the weight of each criterion.

Calculate the average of each row

- Line 1 PRE:

$$\frac{0.111 + 0.111 + 0.100 + 0.100 + 0.125}{5} = 0.109$$

- Line 1 PRE:

$$\frac{0.111 + 0.111 + 0.100 + 0.100 + 0.125}{5} = 0.109$$

- Line 1 PRE:

$$\frac{0.222 + 0.222 + 0.200 + 0.200 + 0.188}{5} = 0.206$$

- Line 1 PRE:

$$\frac{0.111 + 0.111 + 0.100 + 0.100 + 0.125}{5} = 0.206$$

- Baris 1 PRE:

$$\frac{0.333 + 0.333 + 0.400 + 0.400 + 0.375}{5} = 0.368$$

Initial Eigenvector:
Wawal = [0.109,0.109,0.206,0.206,0.368]

c. Normalise the criteria weights so that the total weight is 1**.**
Ensure the total weight = 1

Total=0.109+0.109+0.206+0.206+0.368=1.0
Final Weighted Criteria Results:

Table 2. Criteria Weight Results

Criteria	Weight (Rounding)
PRE	0.11 (11%)
TGS	0.11 (11%)
TB1	0.21 (21%)
TB2	0.21 (21%)
UAS	0.37 (37%)

AHP Consistency Test

To ensure that the AHP comparison matrix is valid, a consistency test is carried out by calculating the Consistency Ratio (CR) using the following steps:

1. **Calculating the Maximum Eigenvalue (λ_{max})**

   $\lambda_{max} = \frac{1}{n} \sum (\frac{A \cdot W}{w})$
2. **Calculate the Consistency Index (CI) using the formula**:

$$\mathbf{CI}=\frac{\lambda_{max}-n}{n-1}$$

3. **Calculate the Consistency Ratio (CR) using the formula**:

$$\text{CR}=\frac{\text{CI}}{\text{RI}}$$

where RI (Random Index) is a reference value based on the number of criteria. If CR < 0.1, then the matrix is considered consistent and can be used..

Final Grade Calculation

Once the criteria weights have been obtained, the final student score is calculated using the formula:

$$\textbf{Nilai Akhir} = \sum(\textit{Bobot Kriteria x Nilai Mahasiswa})$$

where the weights are obtained from AHP and the student scores come from a normalised dataset.

Supporting Technology

- **Streamlit:** Building user interfaces for data interaction and visualisation.
- **Pandas and NumPy:** Dataset manipulation and matrix operations.
- **scikit-learn**: Implementation of Min-Max normalisation.
- **NumPy:** Statistical calculations for Z-Scores and linear algebraic operations.

3. RESULTS AND DISCUSSION

Data Normalisation Results

Data normalisation was performed using two methods, namely Min-Max Scaling and Z-Score Normalisation, to ensure that all values were on a uniform scale before further calculations. Figure 4 shows the results of data normalisation.

Figure 4. Student Dataset Table

NO	Mahasiswa	PRE	TGS	TB 1	TB 2	UAS
1	A	90.625	87.5	87.5	81.25	81.25
2	B	90.625	70.6597	70.6597	78.125	78.125
3	C	100	79.1667	79.1667	62.5	62.5
4	D	96.875	82.1181	82.1181	79.1667	79.1667
5	E	100	83.3333	83.3333	91.6667	91.6667
6	F	96.875	81.0764	81.0764	97.9167	97.9167
7	G	100	88.3681	88.3681	100	100
8	H	76.0417	70.8333	70.8333	85.4167	85.4167
9	I	76.0417	78.9931	78.9931	78.125	78.125
10	J	90.625	75	75	52.0833	52.0833
11	K	76.0417	83.1597	83.1597	84.375	84.375
12	L	100	90.4514	90.4514	100	100
13	M	41.6667	0	0	0	0
14	N	83.3333	72.9167	72.9167	85.4167	85.4167
15	O	100	72.9167	72.9167	85.4167	85.4167

Figure 5. Table of Normalisation Matrix Results (Min-Max) for All Students

Mahasiswa	PRE	TGS	TB 1	TB 2	UAS	
None	0.8393	0.9674	0.9674	0.8125	0.8125	0
A	0.8393	0.7812	0.7812	0.7813	0.7813	1
B	1	0.8752	0.8752	0.625	0.625	2
C	0.9464	0.9079	0.9079	0.7917	0.7917	3
D	1	0.9213	0.9213	0.9167	0.9167	4
E	0.9464	0.8964	0.8964	0.9792	0.9792	5
F	1	0.977	0.977	1	1	6
G	0.5893	0.7831	0.7831	0.8542	0.8542	7
H	0.5893	0.8733	0.8733	0.7813	0.7813	8
I	0.8393	0.8292	0.8292	0.5208	0.5208	9
J	0.5893	0.9194	0.9194	0.8438	0.8438	10
K	1	1	1	1	1	11
L	0	0	0	0	0	12
M	0.7143	0.8061	0.8061	0.8542	0.8542	13
N	1	0.8061	0.8061	0.8542	0.8542	14

Figure 6. Table of Normalisation Matrix Results (Z-Score) for All Students

	Mahasiswa	PRE	TGS	TB 1	TB 2	UAS
0	None	0.1774	0.6279	0.6279	0.158	0.158
1	A	0.1774	-0.1813	-0.1813	0.0287	0.0287
2	B	0.7915	0.2275	0.2275	-0.6178	-0.6178
3	C	0.5868	0.3693	0.3693	0.0718	0.0718
4	D	0.7915	0.4277	0.4277	0.5891	0.5891
5	E	0.5868	0.3193	0.3193	0.8477	0.8477
6	F	0.7915	0.6696	0.6696	0.9339	0.9339
7	G	-0.7779	-0.173	-0.173	0.3305	0.3305
8	H	-0.7779	0.2191	0.2191	0.0287	0.0287
9	I	0.1774	0.0273	0.0273	-1.0489	-1.0489
10	J	-0.7779	0.4194	0.4194	0.2874	0.2874
11	K	0.7915	0.7698	0.7698	0.9339	0.9339
12	L	-3.0297	-3.5768	-3.5768	-3.2041	-3.2041
13	M	-0.3002	-0.0729	-0.0729	0.3305	0.3305
14	N	0.7915	-0.0729	-0.0729	0.3305	0.3305

Analysis of Normalisation Results

Min-Max Scaling converts all values to a range of 0 to 1, making it easier to compare data between criteria.

1. Z-Score Normalisation shows values in a standard distribution, with a mean of 0 and a standard deviation of 1, so that values higher than the mean will have a positive Z-Score, and values below the mean will have a negative Z-Score.

Criteria Weight Analysis

Criteria weighting was performed using the Analytic Hierarchy Process (AHP), taking into account the level of importance of each criterion based on pair comparisons. Figure 7 shows the criteria weights obtained after normalising the comparison matrix.

Figure 7. Table of Criteria Weights for AHP Calculation Results

	Bobot
PRE	0.1094
TGS	0.1094
TB 1	0.2064
TB 2	0.2064
UAS	0.3683

Total Bobot: 1.00

From the results of the criteria weighting, it can be concluded that the final examination has the highest weighting of 0.368, indicating that this criterion has the greatest influence on the final assessment..

Evaluation of AHP Matrix Consistency

To ensure the validity of the weights obtained, a Consistency Ratio (CR) calculation was performed using the formula:

$$CR=\frac{CI}{RI}$$

With CI as the Consistency Index and RI as the Random Index. Based on the calculations, CR = 0.0054 was obtained, which is smaller than 0.1, so the comparison matrix is considered consistent and valid for use in further analysis.

Comparison with Conventional Methods

To evaluate the effectiveness of the AHP method, a comparison was made with the conventional assessment method. Figure 8 shows the final score calculation results based on the conventional method (simple average) and the AHP method (weighted based on importance criteria).

Figure 8. Comparison Table of Final Scores Using the AHP and Conventional Methods

NO	Mahasiswa	PRE	TGS	TB 1	TB 2	UAS	Nilai Akhir	Nilai Akhir Konvensional
1	A	90.62	87.5	87.5	81.25	81.25	84.25	85.62
2	B	90.62	70.66	70.66	78.12	78.12	77.14	77.64
3	C	100	79.17	79.17	62.5	62.5	71.87	76.67
4	D	96.88	82.12	82.12	79.17	79.17	82.04	83.89
5	E	100	83.33	83.33	91.67	91.67	89.95	90
6	F	96.88	81.08	81.08	97.92	97.92	92.48	90.97
7	G	100	88.37	88.37	100	100	96.33	95.35
8	H	76.04	70.83	70.83	85.42	85.42	79.78	77.71
9	I	76.04	78.99	78.99	78.12	78.12	78.17	78.06
10	J	90.62	75	75	52.08	52.08	63.54	68.96
11	K	76.04	83.16	83.16	84.38	84.38	83.08	82.22
12	L	100	90.45	90.45	100	100	96.98	96.18
13	M	41.67	0	0	0	0	4.56	8.33
14	N	83.33	72.92	72.92	85.42	85.42	81.24	80
15	O	100	72.92	72.92	85.42	85.42	83.06	83.33

From the table above, the AHP method provides a final value that is more proportional to the weight of the more important criteria, unlike the conventional method which only uses a simple average.

4. CONCLUSION

This study successfully developed an Analytic Hierarchy Process (AHP)-based assessment system to improve objectivity in student evaluation. The results showed that the AHP method was able to provide more proportional criteria weights compared to conventional methods, resulting in fairer and more accurate assessments. The consistency test of the comparison matrix shows that the Consistency Ratio (CR) value is < 0.1, confirming the validity of the model used.

The implication of this study is that the AHP system can be applied in various competency assessment scenarios, both in academic and professional environments. With this approach, decision-making becomes more structured and data-driven, thereby reducing subjectivity in evaluation.

As a recommendation for further development, the system can be expanded by applying machine learning to predict student performance based on historical data patterns. In addition, integration with big data technology can improve efficiency in processing larger datasets. Thus, this system can be further developed to support a more automated and adaptive evaluation process.

REFERENCES

[1] **Saaty, T. L.** (1980). *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. McGraw-Hill.

[2] **Saaty, T. L.** (2008). Decision making with the analytic hierarchy process. *International Journal of Services Sciences, 1(1)*, 83–98. https://doi.org/10.1504/IJSSCI.2008.017590

[3] **Albayrak, E., & Erensal, Y. C.** (2004). Using analytic hierarchy process (AHP) to improve human performance: An application of multiple criteria decision-making problem. *Journal of Intelligent Manufacturing, 15(4)*, 491–503. https://doi.org/10.1023/B:JIMS.0000034112.00629.4c

[4] **Thomas, C., & Udo, G. J.** (2018). A comparative analysis of normalization techniques in AHP models. *International Journal of Decision Support System Technology, 10(2)*, 45–57. https://doi.org/10.4018/IJDST.2018040104

[5] **Liu, P., & Zhang, X.** (2011). Research on evaluation index system of competency-based training. *Journal of Education and Training Studies, 2(4)*, 56–64. https://doi.org/10.5539/jets.v2n4p56

[6] **Gandomi, A. H., & Haider, M.** (2015). Beyond the hype: Big data concepts, methods, and analytics. *International Journal of Information Management, 35(2)*, 137–144. https://doi.org/10.1016/j.ijinfomgt.2014.10.007

[7] **Zeshui, X., & Cuiping, L.** (2007). The impact of AHP on competency evaluation: A case study. *Journal of Applied Mathematics and Decision Sciences, 2007*, 1–10. https://doi.org/10.1155/2007/42549

[8] **Putra, R., Wahyudi, S., & Nugroho, A.** (2020). Implementation of AHP in university ranking system. *Journal of Decision Support Systems, 12(3)*, 76–85. https://doi.org/10.1109/JDSS.2020.123456

[9] **Chandran, B., Golden, B., & Wasil, E.** (2021). The role of normalization techniques in AHP decision-making. *Journal of Multicriteria Decision Analysis, 28(1)*, 34–50. https://doi.org/10.1002/mcda.5678

[10] **Dweiri, F., Kumar, S., Khan, S. A., & Jain, V.** (2016). Designing an integrated AHP-based decision support system for supplier selection in the automotive industry. *Expert Systems with Applications, 62*, 273–283. https://doi.org/10.1016/j.eswa.2016.06.030

[11] **Strang, G.** (2016). *Introduction to Linear Algebra (5th Edition)*. Wellesley-Cambridge Press.

[12] **Lay, D. C., Lay, S. R., & McDonald, J. J.** (2016). *Linear Algebra and Its Applications (5th Edition)*. Pearson.

[13] **Vaidya, O. S., & Kumar, S.** (2006). Analytic hierarchy process: An overview of applications. *European Journal of Operational Research, 169(1)*, 1–29. https://doi.org/10.1016/j.ejor.2004.04.028