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Optimization of Transportation Distribution Costs Using Improved Vogel's Approximation Method (IVAM) (Case Study: PT. Sinar Putra Pertam)

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A B S T R A C T

Distribution is the process of distributing goods from source to destination. The main problem with distribution is the inefficiencies in distribution costs. The distribution problem can be solved by transportation methods. The transportation method is carried out in two stages, namely finding the initial solution then looking for the optimal solution. This study uses Improved Vogel's Approximation Method (IVAM) initial solution method and Modified Distribution Method (MODI) as the optimal solution method in distributing 3 Kg LPG gas PT. Sinar Putra Pertam. This study aims to find out how IVAM and MODI work in getting optimal solutions, and compare the results of these calculations with the calculation of 3 Kg LPG gas distribution costs from the company. In this study, there were 2 sources of LPG gas filling which were distributed using a fleet of colt trucks to 15 destinations. The distribution cost is calculated based on fleet fuel consumption. The results of the initial solution calculation show IVAM requires three iterations to get a solution. While the calculation results of the MODI optimization test require one iteration to achieve the optimal solution. The distribution cost using IVAM and MODI is the same amounting to IDR 6,816,230.00 so that it is smaller than the conventional calculation results from the company, which is IDR 9,307,000.00. Based on these results, the transportation method used in this study can save 26.76% of the total distribution cost.

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1. INTRODUCTION

Since 2007 the government of the Republic of Indonesia has made a policy of transferring kerosene subsidies to LPG (Liquid Petroleum Gas) which incidentally is an oil and gas sector commodity produced by PT. Pertamina. LPG products distributed consist of two types of products, namely Bright Gas (size 5.5 Kg and 12 Kg) and subsidized LPG (size 3 Kg) (Subakdo & Nugroho, 2016). According to the

Performance Report of the Director General of Oil and Gas (2022), the oil and gas sector, especially LPG gas, is able to contribute to Indonesia's high income. From this, LPG became a fundamental commodity and controlled the lives of many people in Indonesia. Therefore, PT. Pertamina needs to distribute LPG evenly and widely through outbound channels or producer-consumer supply chains in LPG distribution, namely SPPBE (LPG Bulk Transport and Filling Station), Agents, and Sub Agents or LPG Bases which will later be distributed to household and industrial consumers (Subakdo & Nugroho, 2016). This distribution activity is one of the fundamental activities in this oil and gas company.

Over time, PT Pertamina's LPG distribution has expanded and spread to various regions in Indonesia. Based on data from the Director General of Oil and Gas of the Ministry of Energy and Mineral Resources released in June 2023, the number of agents listed as Pertamina's official distributor network is 5179 units. One of them is PT. Sinar Putra Pertam operating in Bantul Regency. In carrying out its operations, this company must calculate LPG stocks in filling warehouses with the number of existing demands, and determine the amount of LPG that must be distributed to 15 districts throughout Bantul Regency. The request system used by PT. Sinar Putra Pertam is a huge request system, so the majority of the number of requests for each base is quite large. According to Febrianto, et al (2023) stated that one of the causes of LPG gas prices that do not match the highest retail price (Het) is the length of the distribution chain and loading and unloading activities at the base. Based on the Regulation of the Governor of the Special Region of Yogyakarta No. 28 of 2015 concerning the Benchmark Price of 3 Kg LPG, the highest retail price that has been set by the regional government and Pertamina is Rp 15,500. Distribution that is not carefully calculated can cause consumers to be disloyal and will look for other suppliers. Therefore, management in distribution activities here is very important to support the company's performance so that there are no complaints from consumers (Hermanto, et al, 2020). Distribution activities are one of the important factors in influencing product prices in the sales of a company. Distribution can be interpreted as the process of distributing goods or services from a source (producer) to a destination (consumer) (Ibnas, et al, 2019). The main problem with distribution is the inefficiencies in distribution costs due to different distances between destinations and sources. This also happened at PT. Sinar Putra Pertam where the distribution process is based on the number of kilometers traveled by the vehicles used, so that ineffective allocation of LPG distribution will incur large costs. Therefore, it is necessary to optimize to get the optimal solution from these calculations.

Optimization is the process of determining which option, out of multiple options, is optimal while considering desirable criteria. In mathematics, optimization can be formulated as a mathematical programming problem that seeks the maximum or minimum value of an objective function, taking into account existing constraints. The goal of optimization is to achieve the best results by taking into account all relevant and possible factors (Gofur, et all, 2023). Krisna & Sumiati (2023) also explained that optimization has the goal of minimizing effort or maximizing desired profits. Therefore, the decision about optimization is referred to as the optimal decision. One way that can be used to overcome product delivery problems in minimizing product shipping costs is to use transportation methods. The transportation method is a tool to solve the problem of shipping products from various suppliers who have their product stock to be sent to several destination locations optimally (Wahyu, et al, 2021). According to Ganesan & Dheebia (2020), transportation methods play an important role in regulating product distribution operations and can be easily implemented, especially in the manufacturing industry. In general, transportation methods can be notated in canonical mathematical form as follows (Mussafi, 2016):

$$\text{Minimize} \quad c^T x \quad \text{(Objective Function)} \tag{1}$$

$$\text{Subject to} \quad A \geq b \quad \text{(Constraint Function)} \tag{2}$$

$$x \geq 0 \quad \text{(Decision Variables)} \tag{3}$$

With matrix vector notation it can be written as follows: (Siregar, 2022),

$$\min_x \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} s.t. \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{11} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \geq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \geq 0 \tag{4}$$

m = Many types of sources available

n = Activities that use resources

i = Number each type of available source (i = 1, 2, 3, ..., m)

j = Number each type of activity that uses the source (j = 1, 2, 3, ..., n)

x_j = Decision variable for j activity = 1, 2, ..., n

a_{ij} = Source i required to produce each activity output J (I = 1, 2, ..., M; j = 1, 2, ..., N)

b_i = The number of sources i available to allocate to each unit of activity (i = 1, 2, ..., m)

c_j = Cost of each unit of activity j to the value of f(x)

There are two steps in achieving the optimum solution of the transport method. The first step is to find the initial solution, then the next step is to find the optimal solution. The methods commonly used in the initial solution search are North West Corner (NWC), Least Cost Method (LCM), and Vogel's Approximation Method (VAM). Vogel's Approximation Method usually produces a better initial solution than the North West Corner method and the Least Cost Method (Trihudiyatmanto, 2018). However, even though it produces better solutions than NWC and LCM, VAM can produce solutions that are not necessarily optimal (Amaliah et al, 2016; Korukoğlu & Balli, 2011). Therefore, a modification of VAM emerged whose results are considered more optimal or close to optimal, namely the Improved Vogel's Approximation Method (IVAM) method. The selection of methods for this initial solution needs to be considered because the better the initial solution obtained, the fewer iterations required in the optimal test.

The next step is that the initial solution that has been obtained can be optimized with optimal test instruments (Feriza & Murni, 2020). The second step towards an optimal solution usually uses the Stepping Stone Method and Modified Distribution (MODI) (Zulfikarizah, 2004). In addition to these two methods, there is also the Modified Allocation Method (MODA Method) initiated by Dr. R. Murugesan in 2022. The MODA method is a development of MODI and includes iterative methods that can be used to test the optimality of initial basic feasible solutions and also optimize them if they are not optimal for transportation problems. In solving transportation problems, this method is claimed to be quite efficient in finding optimal solutions. MODA differs from the MODI method in that it tests base cells for simpler evaluation and loop determination and saves calculation time.

There is no precise calculation involved in the distribution of LPG gas since, according to the observation, PT. Sinar Putra Pertam's strategy only uses traditional calculations based on driving behaviors to determine distribution routes. This may lead to the computation of monthly fluctuations and suboptimal distribution costs. Because of this, PT. Sinar Putra Pertam will be able to calculate the cost of LPG gas disruption in this study more efficiently by using transportation methods in conjunction with mathematical calculations. Thus, the research entitled "Optimization of Transportation Distribution Costs Using Improved Vogel's Approximation Method (IVAM) (Case Study: PT. Sinar Putra Pertam)" aims to optimize the transportation costs of distributing 3 Kg LPG gas by PT. Sinar Putra Pertam in the rayon area of Bantul Regency.

2. LITERATURE REVIEW

The initial stage of preparation is a literature study as research reference material. The main literature used in this study consisted of 3 books and 3 journals. References to the theory of operations research and transportation methods are taken from the book "Riset Operasi dalam Pendekatan Algoritmis" by Jong Jek Siang, the book "Riset Operasi (*Operation Research*) & Penyelesaian Menggunakan *Software WinQSB*" by Trihudiyatmanto, and the book "Riset Operasi Untuk Ekonomi" by Kurdhi, et al. In addition, reference material related to IVAM and MODI algorithms was obtained from journals entitled "Optimization of Goods Distribution Using Vogel's Approximation Method and Stepping Stone Method" by Ratnasari, "An Improved Vogel's Approximation Method for the Transportation Problem" by Korukoğlu & Balli, and "Application of Improved Vogel's Approximation Method in Minimization of Rice Distribution Costs of Perum BULOG" by Nahar. The research that became a reference in this study was a study conducted by Serdar Korukoğlu and Serkan Balli in 2011. The study entitled "An Improved Vogel's Approximation Method for The Transportation Problem" discusses the development of VAM methods to obtain more efficient initial solutions. The results of the study show that IVAM obtained a more efficient initial solution to large-scale transportation problems in terms of the number of iterations and the results of minimum cost calculations.

Nahar, et al in 2018 also conducted research on the IVAM method entitled "Application of Improved Vogel's Approximation Method in Minimization of Rice Distribution Costs of Perum BULOG". This research was conducted on the transportation problem of Perum Bulog Medan with 5 warehouses and 6 distribution points. The purpose of this study is to determine the optimal solution IVAM, as well as compare the results of the optimal solution with the calculations made by the company. The results of the study show that the solution obtained from the IVAM (Improved Vogel's Approximation Method) method on the transportation problem of Perum Bulog Medan is the most optimal solution in terms of the number of iterations and rice distribution costs.

Ratnasari, Yuniarti, and Purnamasari, 2020 in their research entitled Optimization of Goods Distribution Using Vogel's Approximation Method and Stepping Stone discussed the search for optimal solutions to transportation problems in a case study of distributing 3 kg LPG gas cylinders at PT. Tri True Indigenous. This study discusses a case study of the distribution of 3 kg LPG gas which is a transportation problem and can be solved by transportation methods. The methods used in solving transportation problems in this study are the VAM method to find initial solutions and the Stepping Stone method for optimization tests so that optimal solutions are obtained. The results of the study VAM has an initial solution of IDR 24,353,568.00 so as to save transportation costs as much as 45.9% from the previous transportation costs calculated by the company of IDR 45,000,000.00. Meanwhile, after an optimization test using the Stepping Stone method, the total transportation costs amounted to Rp 24,031,104.00, thus saving transportation costs as much as 46.6% of the total costs calculated by the company. Using some of the earlier research mentioned above, Korukoğlu and Serkan Balli (2011) focused on the development of the VAM method to be a more efficient method. In this study, the comparison between the VAM and IVAM methods on several transportation problems is a metaphor rather than a case study on a real problem. Furthermore, Nahar, et al. (2018) concentrated their investigation on the use of IVAM on real distribution problems, namely on the case study of rice distribution. In addition, Ratnasari, et al. (2020) centered on the issue of distributing 3 Kg LPG gas but using the VAM method only. From some of the literature, this study is intended to explore more deeply the effectiveness of the use of the IVAM method if used in a real case study of 3 Kg LPG gas distribution by PT. Sinar Putra Pertam.

3. RESEARCH METHOD

The research method used is applied research with a quantitative approach. The objects discussed in this study are the cost of product distribution from source to destination, the capacity of each warehouse or production site, and the amount of product demand from each destination at PT. Sinar Putra Pertam. While the data used are the number of warehouses, the number of delivery destinations, the capacity of each warehouse, product demand from each destination, and estimated transportation costs for each product shipment.

In this study, literature studies and field observations were carried out so that the secondary data obtained were valid data as a preparatory stage. Collection of research data by means of interviews and data documentation. Then the data obtained is processed through steps, namely compiling data into the form of transportation tables, checking that the amount of inventory is equal to the amount of demand, making transportation models based on transportation tables, determining the initial fissile solution with the Improved Vogel's Approximation Method (IVAM) method, then testing optimality using the Modified Disribution Method (MODI) method. Here is the IVAM algorithm in calculating the initial solution of the transportation problem.

IVAM Method Algorithm (Korukoğlu & Balli, 2011)

1. Create a Total Opportunity Cost (TOC) matrix obtained from the sum of the Opportunity Cost (OC) rows and Opportunity Cost (OC) columns, where for Opportunity Cost (OC) rows calculate the difference between each row and the smallest value of the row. As for the Opportunity Cost (OC) column, calculate the difference between each column with the smallest value in the column. The TOC matrix table can be seen in Table 1.

Table 1. TOC matrix

		Destination				Supply
		d_1	d_2	...	d_n	
Source	s_1	TOC_{11} x_{11}	TOC_{12} x_{12}	...	TOC_{1n} x_{1n}	a_1
	s_2	TOC_{21} x_{21}	TOC_{22} x_{22}	...	TOC_{2n} x_{2n}	a_2
	...	...	...	...	...	...
	s_m	TOC_{m1} x_{m1}	TOC_{m2} x_{m2}	...	TOC_{mn} x_{mn}	a_m
Demand		b_1	b_2	...	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

2. Calculate the penalty of each row and column. The penalty value is derived from the difference between the smallest value of the row and column and the second smallest value of the same row and column.
3. Choose the largest penalty value.
4. Based on the 3rd step, choose the

smallest transportation cost from the three highest penalties.
5. Repeat steps 2-4 until all allocations are met.
6. Calculate the total transportation cost using the initial transportation cost.

The optimum test algorithm used in this study is MODI, with the following steps.

MODI Method Algorithm (Siang, 2014)

1. At initial fissible completion, add ui column (i=1, 2, ..., m) and vj row (j=1, 2, ..., n)
2. Fill one of the ui rows or vj fields with 0.
3. Fill the ui row and other vj columns with the rule: for each base cell applies the equation ui + vj = cij
4. Fill the remaining cells (non-base cells) with the repair index with the equation

$$kij = cij - ui - vj \tag{3.2}$$

If there are cells with cij – ui – vj values < 0 then it means that the table is not optimal. The table is optimal if for non-base cells, the value of cij – ui – vj ≥ 0, if there is only one cell whose cij – ui – vj value is negative then the table is not optimal and needs to be improved optimization.
5. Revise the table by selecting non-base variables (blank cells) with the minimum cij – ui – vj < 0 values.
6. Fill the cells with as much quantity as possible.
7. Adjust the quantity of xij on other cells in the loop.
8. Check whether the new solution is optimal. If not optimal, return to steps 5-8.

4. RESULT AND DISCUSSION

Research Data

1. **Data on Warehouse Capacity and Demand for 3 Kg LPG Gas**

PT. Sinar Putra Pertam collaborates with 2 SPPBE (LPG Bulk Filling and Transportation Station), namely SPPBE Kosan – PT. Lestari Pelita Graha and SPPBE PT. Nature Narrizqi Airmas. Data on the 3 Kg LPG gas filling capacity at each SPPBE in September 2023 can be seen in Table 2.

Table 2. Warehouse capacity filling in September 2023

No	SPPBE	Capacity (Unit)
1	SPPBE Kosan - PT. Lestari Pelita Graha Wates No.km 19.5, Gembongan, Sukoreno, Sentolo, Kulon Progo, Daerah Istimewa Yogyakarta 55664	40.320
2	SPPBE PT. Alam Narrizqi Airmas Madugondo, Sitimulyo, Piyungan, Bantul, Daerah Istimewa Yogyakarta 55792	14.560
	Total	**54.880**

Data on demand for 3 Kg LPG gas from each destination in September 2023 can be seen in Table 3.

Table 3. Demand from destination in September 2023

No	District	Demand	No	District	Demand
1	Bambanglipuro	2.410	9	Pajangan	1.340
2	Banguntapan	2.860	10	Pandak	2.150
3	Bantul	10.710	11	Pleret	2.050
4	Dlingo	1.530	12	Pundong	3.100
5	Imogiri	2.150	13	Sanden	3.430
6	Jetis	8.820	14	Sewon	7.060
7	Kasihan	5.430	15	Srandakan	350
8	Kretek	1.490			
Total				**54.8**	

2. **Transportation Cost Data from Warehouse to Destination**

Data related to the amount of transportation costs in this study were obtained from interviews with the director of PT. Sinar Putra Pertam. The variables that influence the determination of the distribution cost of each cell are vehicle fuel consumption and mileage between source to destination.

Table 3. Cost of distribution of 3 Kg LPG gas from each source to destination (IDR)

Source	LPG gas distribution area 3 Kg														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15
S1	141	148	119	194	163	146	107	172	121	138	172	160	153	131	155
S2	134	85	100	134	119	100	85	158	124	121	97	136	160	87	180

Information:
S1 = SPPBE Kosan - PT. Lestari Pelita Graha, S2 = SPPBE PT. Alam Narrizqi Airmas, D1 = Bambanglipuro, D2 = Banguntapan, D3 = Bantul, D4 = Dlingo, D5 = Imogiri, D6 = Jetis, D7 = Pity, D8 = Kretek, D9 = Display, D10 = Pandak, D11 = Pleret, D12 = Pundong, D13 = Sanden, D14 = Sewon, and D15 = Srandakan.

Discussion

Based on the data on capacity, demand, and distribution costs in tables 2-4, mathematical models will be created as in equation (1), equation (2), and equation (3).

Objective function:

Minimize
$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad (5)$$

$Z = 141x_{11} + 148\,x_{12} + 119x_{13} + 194x_{14} + 163x_{15} + 146\,x_{16} + 107x_{17} + 172\,x_{18} + 121x_{19} + 138x_{110} + 172x_{111} + 160x_{112} + 153x_{113} + 131x_{114} + 155x_{115} + 134\,x_{21} + 83x_{22} + 100\,x_{23} + 134x_{24} + 119x_{25} + 100x_{26} + 85x_{27} + 158x_{28} + 124x_{29} + 121x_{210} + 97x_{211} + 136x_{212} + 160x_{213} + 87x_{214} + 180x_{215}$

with:
Z = Total distribution cost
c_{ij} = Cost of distribution of goods from source i to destination j
x_{ij} = Many units of goods transported from source i to destination j

Constraint function:

Supply
$$\sum_{i=1}^{m} x_{ij} = a_i,$$

with a_i is in supply to i, i = 1, 2.

Demand
$$\sum_{j=1}^{n} x_{ij} = b_j,$$

with b_j is in demand to j, j = 1, 2, ..., 15.

$x_{ij} \geq 0,$ for all i and j

The next step, the model will be changed into the form of a transport table as follows:

Table 4. LPG gas distribution transportation cost (IDR)

Supply	Demand															Supply
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
S1	141 x_{11}	148 x_{12}	119 x_{13}	194 x_{14}	163 x_{15}	146 x_{16}	107 x_{17}	172 x_{18}	121 x_{19}	138 x_{110}	172 x_{111}	160 x_{112}	153 x_{113}	131 x_{114}	155 x_{115}	40320
S2	134 x_{21}	83 x_{22}	100 x_{23}	134 x_{24}	119 x_{25}	100 x_{26}	85 x_{27}	158 x_{28}	124 x_{29}	121 x_{210}	97 x_{211}	136 x_{212}	160 x_{213}	87 x_{214}	180 x_{215}	14560
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

Based on Table 5 it is known that the amount of demand is equal to the amount of inventory so that the transportation problem is balanced.

1. Completion Using Improved Vogel's Approximation Method (IVAM)

The first step of IVAM (Improved Vogel's Approximation Method) is to balance the amount of supply and the amount of demand if the amount of both is not balanced. As seen in table 6, the number of both has been balanced, which is 54,880 units, so there is no need to balance again. The next step is to calculate the Total Opportunity Cost (TOC) matrix.

Table 5. Opportunity cost rows and columns

Supply	Demand															Supply
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
S1	34 / 141 / 7	41 / 148 / 65	12 / 119 / 19	87 / 194 / 60	56 / 163 / 44	39 / 146 / 46	0 / 107 / 22	65 / 172 / 14	14 / 121 / 0	31 / 138 / 17	65 / 172 / 75	53 / 160 / 24	46 / 153 / 0	24 / 131 / 44	48 / 155 / 0	40320
S2	51 / 134 / 0	0 / 83 / 0	17 / 100 / 0	51 / 134 / 0	36 / 119 / 0	17 / 100 / 0	2 / 85 / 0	75 / 158 / 0	41 / 124 / 3	38 / 121 / 0	14 / 97 / 0	53 / 136 / 0	77 / 160 / 7	4 / 87 / 0	97 / 180 / 25	14560
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

The results of the Opportunity Cost row and column are then added into one cell so that the Total Opportunity Cost (TOC) matrix is obtained as in table 7 below.

Table 6. Total opportunity cost-TOC matrix

Supply	Demand															Supply
	D1	D2	D3	D4S	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
S1	41	106	31	147	100	85	22	79	14	48	140	77	46	66	48	40320
S2	51	0	17	51	36	17	2	75	44	38	14	53	84	4	122	14560
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

The next step is to find the penalty value or the difference between the two lowest transportation costs in rows and columns, then choose the three largest penalty values and if there are the same values then arbitrary from Table 8. Iterasi 1 Penyelesaian IVAM each row and column in the transport table. Of the three largest penalty values, as many units as possible are allocated at the smallest transportation cost. The steps to solve it are as follows:

Table 7. Iteration 1: IVAM completion

Supply	Demand															Supply	Row Penalty
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15		
S1	41	106	31	147	100	85	22	79	14	48	140	77	46	66	48	40320	8
S2	51	0 / 2860	17	51 / 1530	36	17	2	75	44	38	14 / 2050	53	84	4	122	8120	2
Demand	2410	0	10710	0	2150	8820	5430	1490	1340	2150	0	3100	3430	7060	350	54880	
Column Penalty	10	106	14	96	64	68	20	4	30	10	126	24	38	62	74		

Table 8. Iteration 2: IVAM completion

Supply	Demand															Supply	Row Penalty
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15		
S1	41	106	31	147	100 / 2150	85	22	79	14	48	140	77	46	66	48 / 350	37820	8
S2	51	0 / 2860	17	51 / 1530	36	17 / 8120	2	75	44	38	14 / 2050	53	84	4	122	0	2
Demand	2410	0	10710	0	0	700	5430	1490	1340	2150	0	3100	3430	7060	0	54880	
Column Penalty	10	-	14	-	64	68	20	4	30	10	-	24	38	62	74		

Table 9. Iteration 3: IVAM completion

Supply	Demand															Supply	Row Penalty
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15		
S1	41 / 2410	106	31 / 10710	147	100 / 2150	85 / 700	22 / 5430	79 / 1490	14 / 1340	48 / 2150	140	77 / 3100	46 / 3430	66 / 7060	48 / 350	0	-
S2	51	0 / 2860	17	51	36	17 / 8120	2	75	44	38	14 / 2050	53	84	4	122	0	-
Demand	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	54880	
Column Penalty	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		

All rows and columns have been fulfilled, so the final result of the solution can be calculated using Improved Vogel's Approximation Method (IVAM) with the final result table as follows.

Table 10. IVAM completion final results

Supply	Demand															Supply
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
S1	141	148	119	194	163	146	107	172	121	138	172	160	153	131	155	40320
	2410		10710		2150	700	5430	1490	1340	2150		3100	3430	7060	350	
S2	134	83	100	134	119	100	85	158	124	121	97	136	160	87	180	14560
		2860		1530		8120					2050					
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

Based on table 11 above, the next step is to calculate the total cost of distribution using the following equation.

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$Z = 141x_{11} + 148\ x_{12} + 119x_{13} + 194x_{14} + 163x_{15} + 146\ x_{16} + 107x_{17} + 172\ x_{18} + 121x_{19} + 138x_{110} + 172x_{111} + 160x_{112} + 153x_{113} + 131x_{114} + 155x_{115} + 134\ x_{21} + 83x_{22} + 100\ x_{23} + 134x_{24} + 119x_{25} + 100x_{26} + 85x_{27} + 158x_{28} + 124x_{29} + 121x_{210} + 97x_{211} + 136x_{212} + 160x_{213} + 87x_{214} + 180x_{215}$

Z = (141 x 2410) + (148 x 0) + (119 x 10710) + (194 x 0) + (163 x 2150) + (146 x 700) + (107 x 5430) + (172 x 1490) + (121 x 1340) + (138 x 2150) + (172 x 0) + (160 x 3100) + (153 x 3430) + (131 x 7060) + (155 x 350) + (134 x 0) + (83 x 2860) + (100 x 0) + (134 x 1530) + (119 x 0) + (100 x 8120) + (85 x 0) + (158 x 0) + (124 x 0) + (121 x 0) + (97 x 2050) + (136 x 0) + (160 x 0) + (87 x 0) + (180 x 0)

Z = 6.816.230

So, the total cost of distribution using Improved Vogel's Approximation Method (IVAM) is IDR 6,816,230.00

2. **Solving Using Modified Distribution Method (MODI)**

Before testing using MODI, first identify the initial solution that has been obtained. This identification is done by checking whether many base variables are equal to (m + n - 1), where m is many rows and n is many columns. In the case study of PT. Sinar Putra Pertam, the results of this initial solution are qualified because many base variables namely 16 are equal to m + n - 1 (2 + 15 - 1) = 16. The next step is to calculate the optimal solution using MODI with the initial fissile solution using VAM and IVAM. Based on chapters 4.2.2 and chapters 4.2.3 it produces the same initial solution using VAM and IVAM so that it is enough to display 1 table of initial solutions for MODI calculations. Table 12 below is a representation of the results of the calculation of the initial VAM and IVAM solutions.

Table 11. IVAM initial solutions

Supply	Demand															Supply
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
S1	141	148	119	194	163	146	107	172	121	138	172	160	153	131	155	40320
	2410		10710		2150	700	5430	1490	1340	2150		3100	3430	7060	350	
S2	134	83	100	134	119	100	85	158	124	121	97	136	160	87	180	14560
		2860		1530		8120					2050					
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

Table 12. Iteration 1 modified distribution

Supply	Demand															Supply
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
	V1 = 141	V2 = 129	V3 = 119	V4 = 180	V5 = 163	V6 = 146	V7 = 107	V8 = 172	V9 = 121	V10 = 138	V11 = 143	V12 = 160	V13 = 153	V14 = 131	V15 = 155	
S1 U1 = 0	141	148	119	194	163	146	107	172	121	138	172	160	153	131	155	40320
	2410		10710		2150	700	5430	1490	1340	2150		3100	3430	7060	350	
S2 U2 = (-46)	134	83	100	134	119	100	85	158	124	121	97	136	160	87	180	14560
		2860		1530		8120					2050					
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

Table 13. Modified distribution improvement index

kij = cij - ui - vj			
k12	19	k28	32
k14	14	k29	49
k111	29	k210	29
k21	39	k212	22
k23	27	k213	53
k25	2	k214	2
k27	24	k215	71

Table 14. Optimum modified distribution-MODI solution

Supply	Demand															Supply
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	
S1	141	148	119	194	163	146	107	172	121	138	172	160	153	131	155	40320
	2410		10710		2150	700	5430	1490	1340	2150		3100	3430	7060	350	
S2	134	83	100	134	119	100	85	158	124	121	97	136	160	87	180	14560
		2860		1530		8120					2050					
Demand	2410	2860	10710	1530	2150	8820	5430	1490	1340	2150	2050	3100	3430	7060	350	54880

Based on table 15 above, the next step is to calculate the total cost of distribution using the following equation.

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$Z = 141x_{11} + 148\, x_{12} + 119x_{13} + 194x_{14} + 163x_{15} + 146\, x_{16} + 107x_{17} + 172\, x_{18} + 121x_{19} + 138x_{110} + 172x_{111} + 160x_{112} + 153x_{113} + 131x_{114} + 155x_{115} + 134\, x_{21} + 83x_{22} + 100\, x_{23} + 134x_{24} + 119x_{25} + 100x_{26} + 85x_{27} + 158x_{28} + 124x_{29} + 121x_{210} + 97x_{211} + 136x_{212} + 160x_{213} + 87x_{214} + 180x_{215}$

Z = (141 x 2410) + (148 x 0) + (119 x 10710) + (194 x 0) + (163 x 2150) + (146 x 700) + (107 x 5430) + (172 x 1490) + (121 x 1340) + (138 x 2150) + (172 x 0) + (160 x 3100) + (153 x 3430) + (131 x 7060) + (155 x 350) + (134 x 0) + (83 x 2860) + (100 x 0) + (134 x 1530) + (119 x 0) + (100 x 8120) + (85 x 0) + (158 x 0) + (124 x 0) + (121 x 0) + (97 x 2050) + (136 x 0) + (160 x 0) + (87 x 0) + (180 x 0)

Z = 6.816.230

Based on the calculation above, the final solution for the distribution cost of 3 Kg LPG gas in the case study of PT. Sinar Putra Pertam using Modified Distribution (MODI) is IDR 6,816,230.00

The cost of PT. Sinar Putra Pertam distributing 3 kg of LPG gas is calculated manually using driver experience, and it comes to IDR 9,307,000.00 for a month. However, the ultimate computation employing the Modified Distribution (MODI) optimization test in conjunction with the IVAM transportation technique yields an ideal distribution cost of IDR 6,816,230.00. Based on these results, it can be seen that the transportation method used in this study if applied by the company can save IDR 2,490,770.00. or 26.76% of the total distribution cost.

Based on the data from the research findings and the study's discussion, a number of recommendations were made, one of which was that businesses should think about using the Improved Vogel's Approximation Method transportation method to optimize the cost of distributing 3 kg of LPG gas because it is more effective and can lower the distribution costs incurred. Furthermore, the research is restricted to transportation systems that utilize the initial solution of IVAM and the optimal solution of MODI. It is envisaged that additional transportation-related research would make use of different techniques like the MODA (Modified Allocation) Method for the ideal solution and the Max Min Vogel's Approximation Method (MMVAM) and Modified Vogel's Approximation Method (MVAM) for the first solution.

5. CONCLUSION

Based on the research, it was found that the calculation of the initial solution using Improved Vogel's Approximation Method (IVAM) resulted in a cost of IDR 6,816,230.00 with a total of 3 iterations. Meanwhile, the optimization test results using Modified Distribution (MODI) amounted to IDR 6,816,230.00 with 1 iteration to get an optimal solution. The calculation result of the final solution is the same as the initial solution, and there is no change in the allocation of the transport table. So it can be concluded that the initial solution of IVAM in this research case study is optimal. In addition, it can be concluded that the transportation method used in this study if applied by the company can save 26.76% of the total distribution costs or IDR 2,490,770.00.

This research also highlights the opportunities for further research to be better. This research is limited to transportation methods with IVAM initial solution as well as MODI final solution. It is expected that future studies can use methods other than those in this study such as Modified Vogel's Approximation Method (MVAM) or Max Min Vogel's Approximation Method (MMVAM) for initial solutions, as well as MODA or NILA Methods developed by Murugesan for optimal solutions. In addition, this research was applied to a case study of the distribution of 3 Kg LPG gas PT. Sinar Putra Pertam with a transportation problem size of 2x15. The author suggests conducting research on the application of transportation methods in other case studies in order to test more deeply the optimization of methods with a more complex size of transportation problems.

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