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Study of Eigenvalues and Matrix Eigenvectors Using MATLAB: Vibration Systems of Multi-Purpose Vehicle (MPV)

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Abstract

Vehicle vibration is a critical factor influencing both passenger comfort and vehicle performance. In this study, we analyze the multi-degree-of-freedom (MDOF) vibrational behavior of a multi-purpose vehicle (MPV) using matrix eigenvalue and eigenvector methods. The vehicle's dynamics are modeled by developing a set of equations of motion that account for the forces acting on the front and rear tires, car body, and pitch angle. MATLAB is utilized to numerically compute the system's eigenvalues and eigenvectors, representing the natural frequencies and vibration modes of the vehicle, respectively. The analysis focuses on the vehicle's response to a 50 mm displacement at the front tire, simulating the effect of road disturbances. The resulting vibrations in the front and rear tires, car body, and vehicle pitch are illustrated over a 1-second time frame. The findings show that the front tire experiences the largest oscillation amplitude of ±1 mm, while the rear tire exhibits a much smaller displacement of ±0.04 mm. The overall car body displacement reaches a maximum amplitude of ±1.3 mm, indicating partial damping of the front tire vibrations. However, the results reveal that the vehicle's suspension system lacks effective damping, as the vibrations do not decrease over time. This behavior could negatively impact ride comfort and safety, particularly on uneven roads. The study concludes that improvements to the vehicle's suspension system are necessary to enhance damping performance. The presented MATLAB-based approach offers a valuable tool for analyzing and optimizing vehicle vibration systems.

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1. Introduction

In the automotive and machining industries, vibrations are often closely related to engine performance and overall vehicle dynamics. In particular, the suspension system of a vehicle is a complex vibrational system that can be modeled with multiple degrees of freedom (DOF). The suspension system plays a critical role in ensuring vehicle stability during braking, enhancing driving comfort, and minimizing the impact of road noise, vibration, and bumps [1], [2]. Vibrations in vehicles affect both vertical and horizontal movement, especially when the vehicle is exposed to external forces, such as road disturbances [3]. To accurately analyze these vibrations, a Multi-Degree-of-Freedom (MDOF) approach is used, where vibrations are attributed to different mass points in the vehicle [4], [5].

Eigenvalues and eigenvectors play a critical role in solving many differential equations, particularly in engineering and matrix-based research [6], [7]. These mathematical tools are widely applied in fields such as advanced dynamics, quantum mechanics, control theory, vibration analysis, and electric circuits [8]. For instance, Sharma et al. (2017) investigated the dynamic stability of a three-wheeled vehicle by modeling its ride behavior using a 9-degree-of-freedom (DOF) system. Through MATLAB simulations, they employed eigenvalue analysis to determine the vehicle's critical speed and identify the parameters that influence its stability [9].

Reference [10] investigated the natural frequencies and simulated dynamic modes of a four-wheel SUV's suspension system using a half-car model with 4 degrees of freedom (DOF). MATLAB was employed to solve the motion equations and derive the dominant dynamic mode frequencies through eigenvector analysis of the suspension system. In another study, reference [11] analyzed the

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connection and oscillation of brake systems by calculating the variability and pseudo-variability of complex eigenvalues. These eigenvalues were used to determine the statistical properties of both the real and imaginary components, offering insights into the brake system's dynamic behavior.

Reference [1] presents a method for analyzing the pitch and bounce of a vehicle body using a Multi-Degree-of-Freedom (MDOF) approach, considering both damping and stiffness effects. The methodology combines manual computations with simulations using OCTAVE and ANSYS. The study was conducted successfully, providing a detailed analysis of each graphical element, and the results were effectively disseminated, contributing valuable insights into vehicle vibration behavior.

Reference [12] employed MATLAB to analyze the vibrations of a dual-mass mechanical system consisting of a body and a wheel. The objective of the study was to determine the amplitude-frequency characteristics of the system and investigate how different parameters affect the root mean square (RMS) value of the vibration. Building on this and other prior research, the author of this study will focus on eigenvalue and eigenvector matrices using MATLAB [13] to model the vibration system of Multi-Purpose Vehicle (MPV) type cars. The technical data for this analysis is sourced from the guidebook of a specific MPV model, referred to as Car A.

2. Methods

2.1. Materials

The experimental investigation was conducted on a Multi-Purpose Vehicle (MPV), designated as Car A, using detailed technical specifications obtained from the vehicle's guidebook [14]. The model utilized in this study, including dimensions and weight distributions, is critical for accurately replicating the physical characteristics that influence vehicle dynamics under vibrational analysis. The Car A model properties for vibration modeling was captured on the Figure 1 and Table 1.

2.2. Experimental procedure

Road excitation for load conditions will use modeling from bicycle shock vibration as shown in Figure 3 [15]. A Multi-Degree-of-Freedom model was developed to simulate the vibrational dynamics of the vehicle. This model considered various degrees of freedom, including pitch, bounce, and lateral and longitudinal movements of the vehicle. The primary variables analyzed included the vehicle's effective headroom, legroom, shoulder room, and the natural frequencies of the suspension system. The MDOF model was implemented in MATLAB, leveraging its robust computational capabilities to solve the complex differential equations that describe the vehicle's dynamic behavior.

Data for the experiment was derived from both simulated environments and empirical measurements. Road excitation for load conditions was modeled using a standard bicycle shock vibration profile, which provided a controlled yet realistic interaction scenario for the vehicle's suspension system. Key parameters such as the amplitude-frequency characteristics of the system and the root mean square value of vibration were measured.

Eigenvalue and eigenvector analysis was employed to identify the vibrational modes and corresponding natural frequencies of the vehicle. This analysis helped in determining the critical points where the vibrational response of the vehicle could potentially lead to stability issues or discomfort. MATLAB's eigensolver functions were utilized to extract these values from the system's stiffness and mass matrices.

The following are the equations of motion for the vibrating bicycle model of a vehicle.

$$I\ddot{\theta} + (a_1^2 p_1 + a_2^2 p_2)\dot{\theta} + (a_1^2 k_a + a_2^2 k_a)\theta + (a_2 p_2 - a_1 p_1)\dot{x}c + (a_2 k_b - a_1 k_a)x_c + a_1 p_1 \dot{x}_1 - a_2 p_2 \dot{x}_2 + a_1 k_a x_1 - a_2 k_b x_2 = 0 \quad (1)$$

$$m_1\ddot{x}_1 + p_1\dot{x}_1 + (k_{ta} + k_a)x_1 + a_1 p_1 \dot{\theta} + a_1 k_a \theta - \dot{p_1} x_c - k_a x_c = 0 \quad (2)$$

$$m_2\ddot{x}_2 + p_2\dot{x}_2 + (k_{tb} + k_b)x_2 + a_2 p_2 \dot{\theta} + a_2 k_b \theta - \dot{p_2} x_c - k_b x_c = 0 \quad (3)$$

$$m_c\ddot{x}_c + \{(p\}_2 + p_1)\dot{x}_c + (k_a + k_b)x_c + (a_2 p_2 - a_1 p_1)\theta + (a_2 k_b - a_1 k_a)\theta - p_1\dot{x}_1 - p_2\dot{x}_2 - k_a x_1 - k_b x_2 = 0 \quad (4)$$

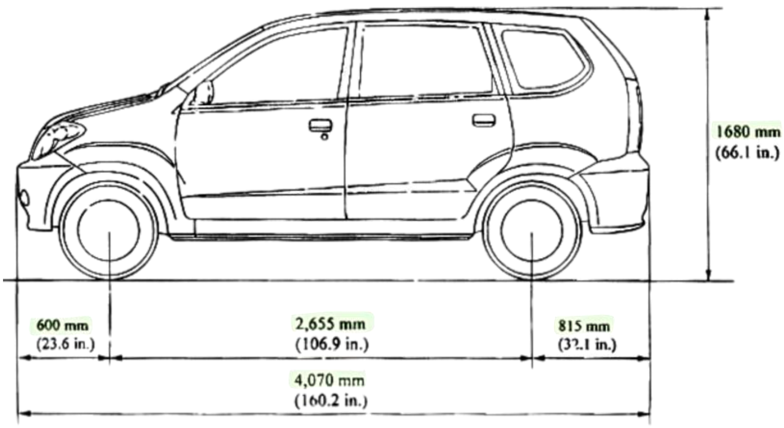

Figure 1. Dimensions of Car A

Table 1. Major technical specification of Car A

Specification	Measurement (mm/inches)	Measurement (kg/lb)
Model code	F601RM-GMJDEJ	
Overall length	4070 (160.2)	
Overall width	1680 (66.1)	
Overall height	1685 (66.3)	
Wheel base	2655 (104.5)	
Tread front	1405 (55.3)	
Tread rear	1415 (55.7)	
Effective head room front	997 (39.3")	
Effective head room rear	876 (34.5")	
Effective leg room front	1030 (39.8")	
Effective leg room rear	970 (39.1")	
Shoulder room front	1400 (55.1)	
Shoulder room rear	1400 (55.1)	
Overhang front	600 (23.6)	
Overhang rear	815 (32.1)	
Min. Running ground clear-	190 (7.5")	
Angle of approach	35°	
Angle of departure	25.1°	
Curb weight front		535 (1,179)
Curb weight rear		520 (1,146)
Curb weight total		1,055 (2,326)
Gross vehicle weight front		625 (1,378)
Gross vehicle weight rear		860 (1,896)
Gross vehicle weight total		1,485 (3,274)
Fuel tank capacity		45 (9.9)
Luggage capacity	0.151 (5.3) cu.ft.	

3. Results and Discussion

3.1. Matrix of displacement vector, mass, stiffness, damping

The matrices derived from the vehicle's dynamic model provide essential insights into its behavior under vibrational loads. The matrices M, P, and K are fundamental to understanding how the vehicle responds to various forces and vibrations.

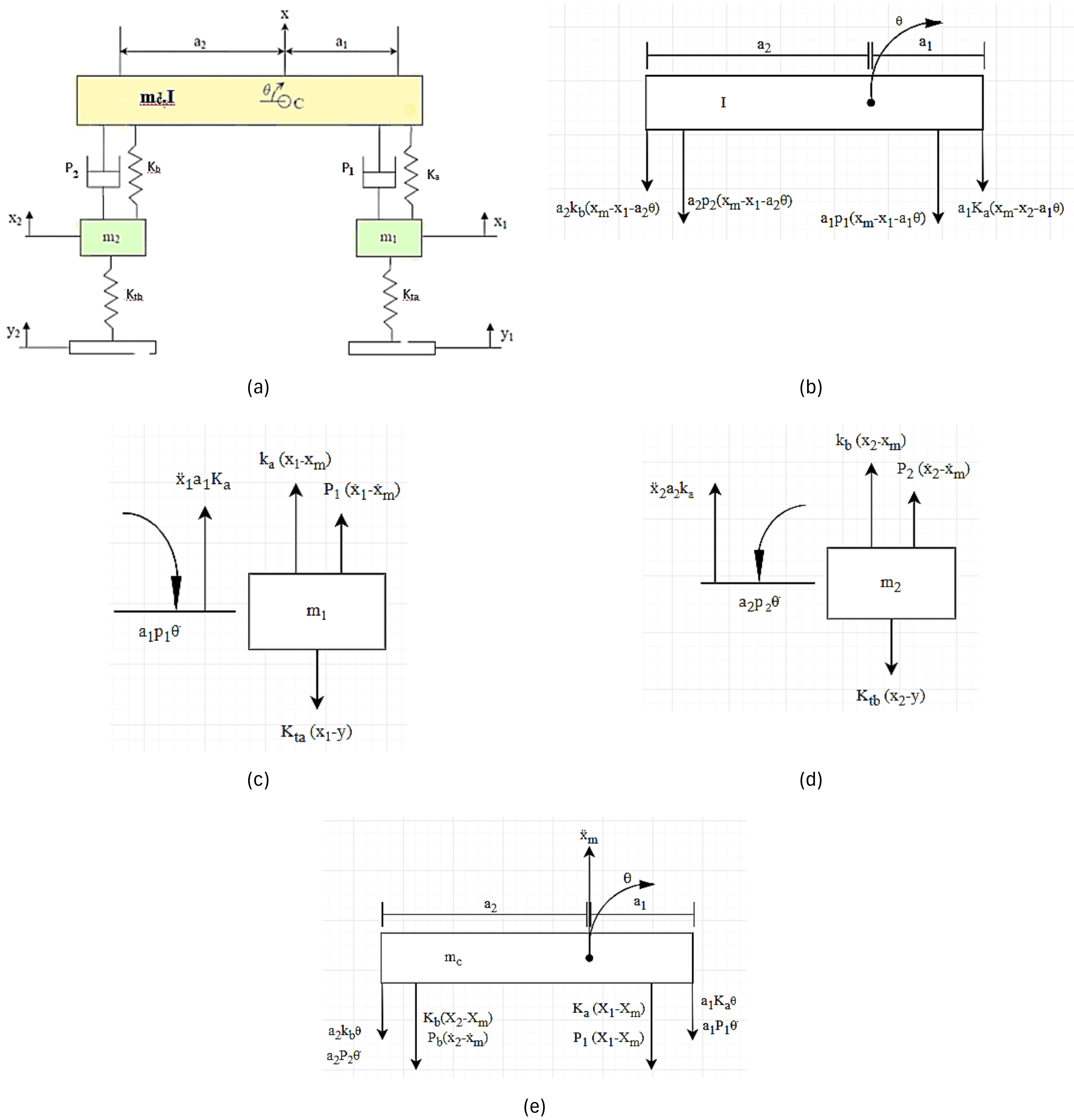

Figure 2. Vehicle bicycle model with 4 degrees-of-freedom (a) Free body diagram of car rational (b) Free body diagram of rear tire (c) Free body diagram of front tire (d) Free body diagram of car translation (e) Free body diagram of car translation

Equation (5) represents the mass matrix (M), where I is the moment of inertia, m_1 and m_2 are the masses of different components, and mc is the mass of the car's body. This matrix is crucial for analyzing the distribution of mass within the vehicle and its impact on the natural frequencies of vibration.

$$M = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_c \end{bmatrix} \tag{5}$$

$$P = \begin{bmatrix} p_1 + p_2 & a_2p_2 - a_1p_1 & -p_1 & -p_2 \\ a_2p_2 - a_1p_1 & p_1a_1^2 + p_2a_2^2 & a_1p_1 & -a_2p_2 \\ -p_1 & a_1p_1 & p_1 & 0 \\ -p_2 & -a_2p_2 & 0 & p_2 \end{bmatrix} \tag{6}$$

$$K = \begin{bmatrix} k_a + k_b & a_2k_b - a_1k_a & -k_a & -k_b \\ a_2k_b - a_1k_a & k_aa_1^2 + k_ba_2^2 & a_1k_a & -a_2k_b \\ -k_a & a_1k_a & k_a + k_{ta} & 0 \\ -k_b & -a_2k_b & 0 & k_b + k_{tb} \end{bmatrix} \tag{7}$$

Equation (6) outlines the damping matrix (P), indicating the damping coefficients such as p_1 and p_2 which help in damping out vibrations. The terms involving a_1 and a_2 likely represent positional parameters affecting how damping is distributed across the vehicle.

Equation (7) is the stiffness matrix (K), which plays a vital role in defining how the vehicle's structure resists deformations due to applied loads. The elements ka and kb are the stiffness coefficients, whereas k_{ta} and k_{tb} could represent additional stiffness contributions from the tires or other components.

These matrices are left symbolic, but they make use of the variable data from Figure 2. where ω is the I natural frequency, $\widetilde{K}$ is the mass normalized stiffness matrix, and I is the spectral matrix of $\widetilde{K}$. The system's natural frequencies are obtained by taking the square root of the eigenvalue diagonal after determining the eigenvalues.

$$\widetilde{K} = \frac{1}{\sqrt{M}}\frac{1}{\sqrt{KM}} \tag{8}$$

$$P^T\widetilde{K}P = \Lambda \tag{9}$$

$$\Lambda = \text{diag}\ (\lambda) = \text{diag}\ (\omega^2) \tag{10}$$

$$S = \frac{P}{\sqrt{M}} \tag{11}$$

3.2. Result of MATLAB from eigenvalue and eigenvector

For each equation of motion, the vibration responses to different figures are shown. The front tire, rear tire, car pitch, and car bounce all correlate to the numbers. Plotting and recording are done within a 1.0-second analytical time period. The MATLAB-generated figure below illustrates the system under initial conditions of a 50 mm displacement on the front tire unsprung mass.

Figure 4 illustrates the front tire vibration motion of a vehicle, providing a time-domain representation of the displacement in meters over a one-second interval. This graph is critical for understanding how the front tire behaves under dynamic conditions.

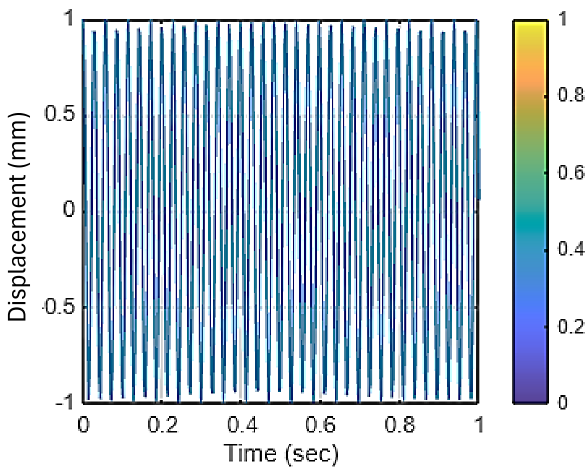

Figure 3. Front tire vibration motion

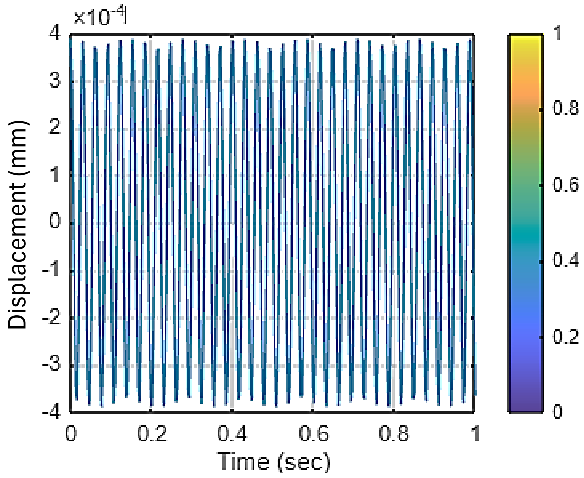

Figure 4. Rear tire vibration motion

The plot shows a highly periodic oscillatory motion, indicating that the tire experiences regular vibrations, as shown in Figure 4. The front tire of the car has the largest bounce oscillation, which is +/- 1 mm. This means that the tire moves up and down with a maximum amplitude of 1 mm. This indicates that the front tire experiences quick, repetitive movements which are significant for understanding the impact transmission from the road to the tire. Persistent high-amplitude oscillations, like those shown, can adversely affect the vehicle's ride quality and handling stability. For passenger comfort and vehicle safety, it is crucial to control such vibrations.

Figure 5 presents the rear tire vibration motion over a one-second period, plotted with displacement in millimeters against time. The rear tire exhibits high-frequency oscillations, indicated by the tightly packed waveform. The amplitude of these oscillations is relatively small, with a peak-to-peak range around ±0.04 mm. The consistency of the amplitude throughout the graph implies minimal damping specific to the rear tire. This could indicate that the rear suspension is designed to maintain stability rather than absorb vibrations, relying on the front suspension to manage the majority of the kinetic energy from road impacts.

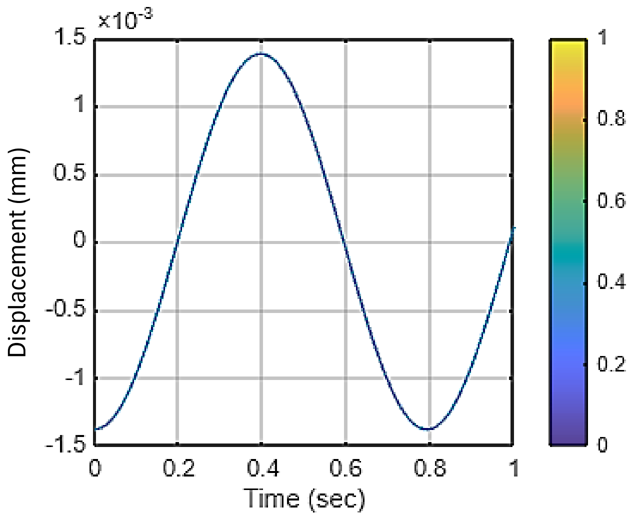

Figure 5. Car vibration motion

Figure 6 illustrates the overall car body vibration motion, measured over a one-second period and plotted as displacement in millimeters. The graph shows a clear, single oscillatory cycle within the second, suggesting a lower frequency compared to the tire-specific vibrations. The amplitude of the oscillation peaks at approximately ±1.3 mm, which is a moderate displacement indicating that the car body is relatively stable under the given conditions. The fact that the car body exhibits a larger displacement than the rear tires but less than the front tires suggest a gradation in how vibrations are absorbed and dampened along the vehicle's length. The displacement curve demonstrates a relatively smooth sinusoidal form, which implies that the car's suspension system is effectively absorbing and then slowly releasing the vibrational energy. The lack of multiple oscillatory peaks indicates good damping properties, as the system returns to equilibrium without excessive rebounding or residual oscillations.

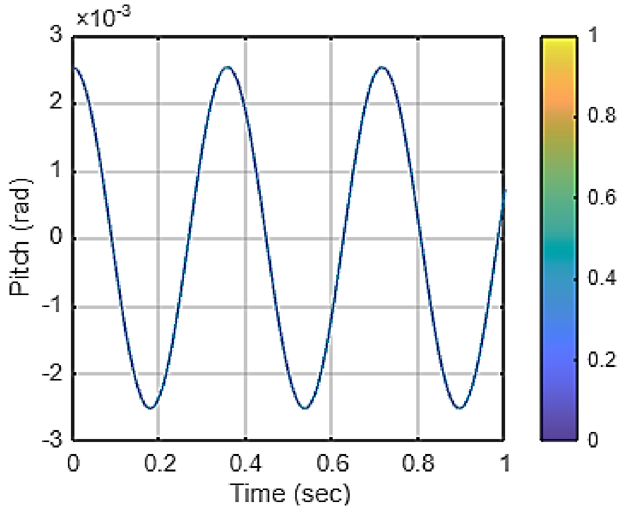

Figure 6. Car rotation motion

Figure 7 displays the rotational motion of the car measured in pitch (radians) over a time period of one second. The pitch amplitude is relatively small, peaking at approximately ±0.0025 radians (or about ±0.02 degrees). Despite its small magnitude, the consistency and periodic nature of these rotations highlight the vehicle's sensitivity to pitch dynamics. The absence of a decrease in amplitude across the observed period suggests inadequate damping in the vehicle's pitch dynamics. Ideal damping would gradually reduce the amplitude of pitch oscillations, leading to a smoother ride and better handling. Although the observed pitch amplitudes are small, the frequency and persistence of these oscillations can impact the vehicle's overall stability and comfort. Persistent pitch oscillations, particularly at higher speeds or in response to abrupt driving maneuvers, could compromise handling and ride quality.

Further dynamic testing should be conducted under varied driving conditions to evaluate the effectiveness of any modifications. Simulation models could also be employed to predict the outcomes of different suspension setups on pitch dynamics. Based on the discussion above, the current car suspension system is indicated not to have good damping. This can cause the car to feel less comfortable and vibrate more when passing through uneven roads. It is recommended to carry out further checks on the car's suspension components to find out the cause of the absence of damping.

4. Conclusions

This study utilized eigenvalues and eigenvectors calculated from the mass and stiffness matrices to determine the natural frequencies and vibration shape modes of a vehicle, highlighting differential vibrational responses across the vehicle's structure. The analysis revealed that while the front tires exhibited the largest vibration amplitude of approximately ±1 mm, the rear tires showed significantly lower vibrations at ±0.04 mm, and the car body itself displayed a maximum amplitude of ±1.3 mm. Despite some damping effectiveness in reducing front tire vibrations, the overall suspension system did not adequately attenuate vibrations over time, indicating limited damping efficiency. Additionally, the vehicle's minor rotational response of ±0.02 degrees, although small, was noted as crucial for driving comfort. The use of MATLAB for simulation and visualization was instrumental in understanding how changes in stiffness and damping coefficients impacted the vehicle structure. The study suggests that enhancements in suspension technology could further improve vehicle performance and passenger comfort, recommending further investigation into advanced damping mechanisms and continuous performance monitoring under varied operational conditions.

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