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Heat Distribution Simulation in a Square Aluminum 7075 Plate Using Laplace Equation and MATLAB

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Abstract

The efficient management of heat transfer from aircraft engines to the wings is vital for maintaining thermal efficiency and structural integrity in modern aircraft design. Excessive heating of the wings, caused by engine-generated heat, can negatively impact aerodynamic performance and safety. This study focuses on analyzing heat distribution in a square aluminum 7075 plate to better understand heat transfer mechanisms. Using the Laplace equation, implemented through MATLAB (2023 Online Version), we aim to simulate and analyze heat distribution on the plate. The numerical method employed in this research involves solving the Laplace equation with Neumann boundary conditions, which represent insulated edges. The Liebmann method is used to iteratively reduce error to less than 1%. Simulations are conducted on an aluminum 7075 plate of dimensions $4x10^{-2}$ m x $4x10^{-2}$ m under various temperature conditions at the edges. Numerical results show that at the 9th iteration, the error reaches 0.71%, while MATLAB simulations yield an error of 0.4681% at the same iteration. The heat distribution across the plate is clearly visualized, and the analysis indicates that increasing the number of grids improves both the clarity and accuracy of the simulation results. In conclusion, this study demonstrates that applying the Laplace equation via MATLAB is an effective approach for analyzing heat distribution in aluminum 7075 plates. The results show that a finer grid resolution enhances accuracy, with a 101-grid system providing particularly clear and precise heat distribution patterns. These findings contribute to the optimization of thermal system designs, especially in aviation-related applications.

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1. Introduction

Efficient heat transfer in aircraft plays a critical role in enhancing thermal performance and preserving structural integrity. Aircraft engines generate substantial heat during operation, making heat management essential in modern aircraft design. The wings, as one of the main structural components, are particularly vulnerable to excessive heating, which can impair aerodynamic performance and compromise flight safety. These challenges necessitate a deep understanding of heat transfer mechanisms and the development of effective technological solutions [1].

In aircraft design, heat distribution in aluminum materials is especially important due to aluminum's favorable mechanical properties, such as high tensile strength and an excellent strength-to-weight ratio. Aluminum, particularly the 7075 alloy, is commonly used in aircraft structures like wings for its ability to withstand heavy loads while remaining lightweight. However, its relatively high thermal conductivity makes it critical to manage heat distribution effectively, as improper heat management can lead to issues such as thermal deformation and material fatigue, which may jeopardize flight safety.

Although previous studies have explored aspects of heat transfer in aviation, further research is necessary to develop a comprehensive understanding of optimal heat distribution and strategies to prevent overheating. This study aims to address this gap by analyzing heat distribution in aluminum 7075 plates, focusing on the heat transfer mechanisms from aircraft engines to wings [2]. Mathematical models are required to explore the properties of heat transfer, often utilizing numerical methods like elliptic partial differential equations, which can be solved through computational techniques.

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The heat distribution phenomenon on a square plate is used to describe temperature distribution within a given domain. Lukman Samatowa et al. demonstrated that temperature distribution in steady-state heat flow models on metal plates can be accurately represented using Microsoft Excel software [3]. Key factors influencing the accuracy of temperature distribution include grid size selection, boundary conditions at the plate edges, and internal heat sources. Imam Noor et al. (2020) further explained that temperature distribution in thin brass plates is higher compared to thin iron plates [4]. Yaochuang Han et al. highlighted the dependency of numerical results for the Stokes flow equation on Gaussian quadrature [5], while Rachmawati demonstrated the application of MATLAB for solving partial differential equations [6].

In summary, previous research has shown that software tools like Microsoft Excel can model temperature distribution on metal plates with accuracy, with factors such as grid size and boundary conditions being critical [3]. Variations in heat distribution between different materials, such as brass and iron, have been observed [3]. Numerical methods remain vital for accurate analyses [5], and MATLAB has proven useful for solving partial differential equations [6]. Nevertheless, there is a lack of research on heat distribution in aluminum 7075 for aviation applications using the Laplace equation with MATLAB. This study seeks to address this gap by performing a comprehensive heat distribution analysis on aluminum 7075 square plates using the Laplace equation in MATLAB.

The goal of this research is to deepen the understanding of heat distribution on square aluminum 7075 plates by applying the Laplace equation through MATLAB simulations. It is anticipated that the findings will provide valuable insights into heat transfer characteristics, contributing to the design and optimization of more efficient and thermally secure aircraft. The results of this study may also inform the development of advanced thermal system designs across various engineering and technological fields, particularly for aircraft wing materials.

2. Methodology

This research employs a numerical method, specifically the Laplace equation, facilitated by MATLAB Online Version 2023, to model heat distribution. MATLAB offers a powerful and flexible environment for solving partial differential equations and other numerical problems.

2.1. Numerical Methods

In this study, Neumann boundary conditions are applied at the plate edges to simulate insulation and modify heat transfer with the surrounding environment. This approach creates a simulation environment that closely reflects real-world conditions.

Additionally, the Liebmann method is used for numerical solutions to iteratively achieve an error of less than 1% [7]. The Liebmann method is a simple yet effective numerical technique for solving the Laplace equation, making it particularly well-suited for analyzing heat distribution in materials like aluminum 7075 [8]. Its advantages include ease of implementation, efficient memory usage, and stable convergence, which can be further improved with relaxation parameters. Moreover, the method is highly flexible, capable of accommodating various boundary conditions and complex geometries. These features make the Liebmann method a reliable tool for heat transfer modeling, contributing to the development of more efficient and accurate thermal designs in engineering applications, including those in the aviation industry.

The use of MATLAB further enhances this research by providing speed, accuracy, and flexibility in applying numerical methods to solve the Laplace equation [9]. This combination of tools and techniques supports robust heat transfer analysis, aiding in the design of advanced thermal systems.

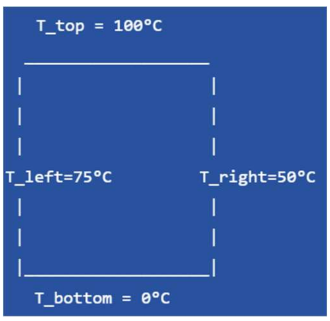

Figure 1. Computational domain diagram with applied boundary conditions

2.2. Boundary Conditions

The boundary conditions applied in this research are highly relevant to real-world scenarios. Neumann boundary conditions, which describe edges with constant or zero temperature gradients, are ideal for modeling thermally insulated parts of aircraft wings to prevent heat loss. On the other hand, Dirichlet boundary conditions, which fix the temperature at the plate edges, represent parts of the wing in direct contact with the engine or other heat sources, providing a realistic depiction of heat distribution within the aircraft structure. By incorporating these boundary conditions, the simulations offer a more accurate and practical representation, delivering crucial insights for the design of thermally efficient and safe aircraft systems. Figure 1 shows a diagram of the computational domain with the applied boundary conditions.

The heat distribution process is modeled on a 7075-aluminum plate with dimensions of 4×10^{-2} m x 4×10^{-2} m [10]. Initially, the temperature at the plate's boundaries is set as follows: 50°C on the right, 75°C on the left, 100°C at the top, and 0°C at the bottom. This setup assumes that the bottom edge is insulated, maintaining a constant temperature, as shown on Figure 2. The plate is heated to a certain temperature with λ=1.5 and $\mathcal{E}_s$=1% based on Liebman method. The temperature measurements are taken at specific points along the four edges of the plate, as shown in Figure 3. The research follows several key steps, including establishing a mathematical model for temperature distribution on the thin plate, converting the analytical equations into a numerical format using the finite difference method for the Laplace equation, performing computational calculations using MATLAB, analyzing the resulting heat distribution profiles, and compiling the findings in a comprehensive report [11].

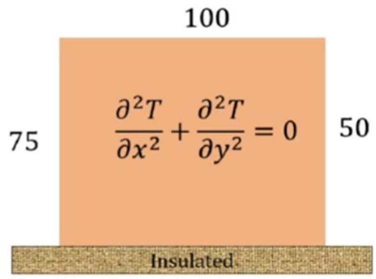

Figure 2. Aluminum 7075 thin plate elements are in equilibrium

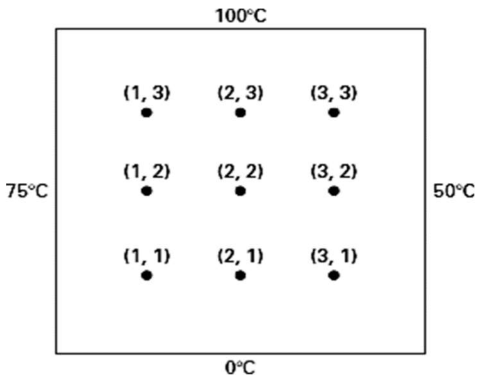

Figure 3. Boundary conditions and grids in the model

3. Results and Discussion

The heat distribution simulation in this study employs the Liebmann method to iteratively solve the Laplace equation for temperature distribution [12]. The numerical solution is presented below.

For 1st iteration

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4} \tag{1}$$

$$T_{11} = \frac{0 + 75 + 0 + 0}{4} = 18{,}75$$

$$\mathrm{T}_{i,j}^{\text{new}} = \lambda \mathrm{T}_{i,j}^{\text{new}} + (1-\lambda)\mathrm{T}_{i,j}^{\text{old}} \tag{2}$$

$$\mathrm{T}_{11} = 1.5 \times 18.75 + (1-1.5) \times 0 = 28.125$$

For i=2, j=1

$$T_{21} = \frac{0 + 28{,}125 + 0 + 0}{4} = 7{,}03125$$
$$T_{21} = 1{,}5(7{,}03125) + (1-1{,}5)0 = 10{,}54688$$

For i=3, j=1

$$T_{31} = \frac{50 + 10{,}54688 + 0 + 0}{4} = 15{,}13672$$
$$T_{31} = 1{,}5(15{,}13672) + (1-1{,}5)0 = 22{,}70508$$

Since all initial temperatures $T_{i,j}$ start from zero, the relative error ε_a for all points in the first iteration is 100%. The error for $T_{1,1}$ is calculated as follows:

$$(\varepsilon_a)_{1,1} = \left|\frac{32{,}51953 - 28{,}125}{32{,}51953}\right| 100\% = 13{,}5\%$$

Since this error exceeds the threshold of 1%, the iteration process continues until the error falls below the acceptable value (<1%). After several iterations, the error decreases gradually. In the 9th iteration, the error for $T_{1,1}$ is reduced to 0.71%, which satisfies the convergence criterion of less than 1%. The error calculation for $T_{1,1}$ in the 9th iteration is shown as:

$$(\varepsilon_a)_{1,1} = \left|\frac{43{,}000 - 42{,}995}{43{,}000}\right| 100\% = 0{,}71\%$$

This result indicates that the temperature distribution has converted to a stable solution within the desired accuracy.

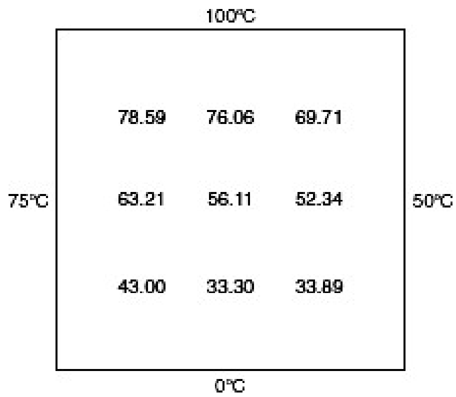

Figure 4. Heat distribution on the plate as a result of numerical calculations

From Figure 4, the results of the numerical calculations on the heated plate with specified boundary conditions—100°C on the top edge, 50°C on the right edge, 75°C on the left edge, and 0°C on the bottom edge—demonstrate the effectiveness of the Liebmann method. The error value steadily decreases over the iterations, falling below 1% at the 9th iteration, with a final error of 0.71%. When using MATLAB for the same simulation, the error at the 9th iteration is recorded as 0.4681%, highlighting MATLAB's efficiency in achieving more precise results under the same conditions.

3.2. Simulation results on MATLAB

The MATLAB simulation results demonstrate how the temperature distribution on the plate evolves over the iterations, converging towards a solution with an error of less than 1%—specifically 0.4681%—by the 9th iteration. The boundary conditions, set as 100°C at the top, 50°C at the right, 75°C at the left, and 0°C at the bottom, dictate how heat spreads across the plate.

The simulation results using MATLAB reveal a reliable temperature distribution across the aluminum plate, with accurate convergence after only 9 iterations, as shown in Table 1.

Table 1. Results of the 9th iteration calculation using MATLAB

T node	1	2	3	4	5	6	7	8	9
Iteration 1	100	100	100	100	100	100	100	100	100
Iteration 2	75	82,7477	82,0073	77,6509	68,5038	48,8089	0	0	50
Iteration 3	75	72,4816	67,0357	59,1536	46,8148	28,4867	0	0	50
Iteration 4	75	64,2861	54,1658	43,6365	32,3539	17,9629	0	0	50
Iteration 5	75	54,4997	40,2657	30,2727	21,1351	11,0964	0	0	50
Iteration 6	75	37,7015	23,6722	16,1141	10,6315	5,4196	0	0	50
Iteration 7	75	0	0	0	0	0	0	0	50
Iteration 8	75	0	0	0	0	0	0	0	50
Iteration 9	0	0	0	0	0	0	0	0	0

3.3. Heat distribution results analysis on grid model variance

The analysis of heat distribution simulations using different grid resolutions (5, 81, and 101 grids) provides valuable insights into the accuracy and detail of thermal modeling. In the simulation with 5 grids, as shown in Figure 5, the temperature distribution is coarse, providing a basic approximation of the heat flow across the plate. With fewer grid points, the model lacks detail, and the temperature transitions between boundary conditions are less smooth. This grid size might be useful for initial testing or quick estimations but lacks precision.

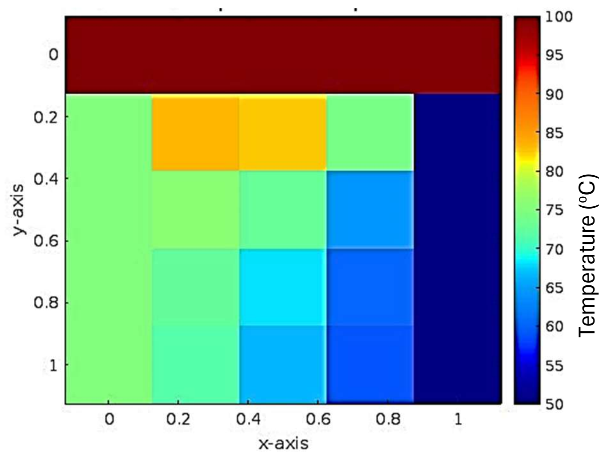

Figure 5. Heat distribution result on a 5 grids model

As the number of grids increases to 81, as shown in Figure 6, the simulation provides much more detail and accuracy in the heat distribution. The temperature gradients are smoother, and the results more closely approximate the actual heat distribution across the plate. This grid size is sufficient for many engineering applications where moderate accuracy is required, as it strikes a balance between computational cost and precision.

With 101 grids, as shown in Figure 7, the heat distribution is even more refined, showing a highly detailed and smooth transition between temperatures across the plate. This finer grid resolution enhances the accuracy of the simulation, allowing for more precise predictions of how heat will flow from the high-temperature regions (e.g., near the engine) to the cooler parts of the plate. The use of a 101-grid system is ideal for applications where high accuracy is essential, such as in the thermal design of aircraft components.

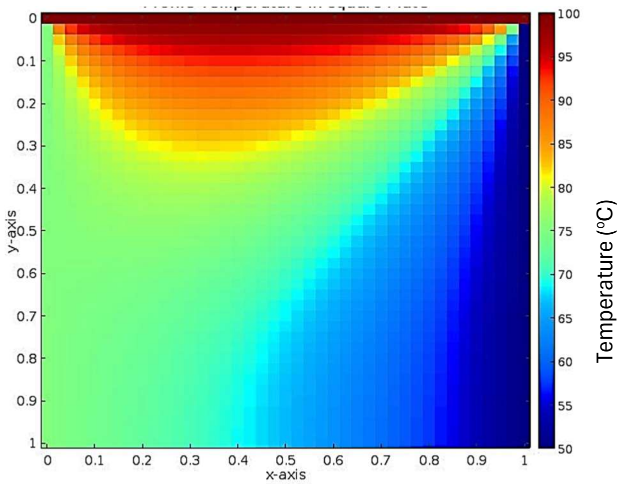

Figure 6. Heat distribution result on a 81 grids model

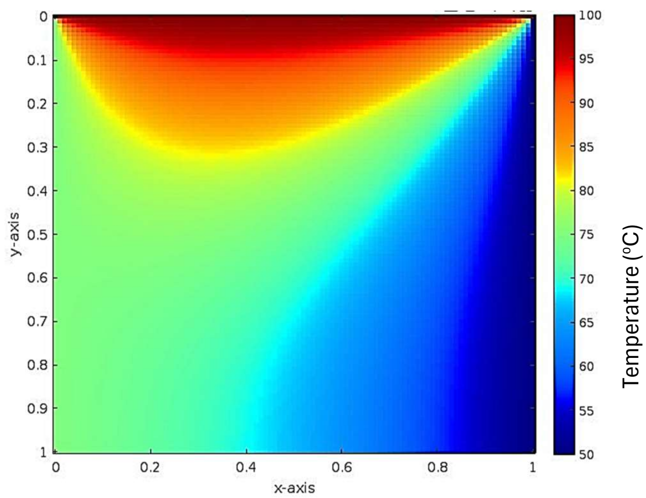

Figure 7. Heat distribution on a 101 grids model

The simulation results using MATLAB demonstrate high consistency with similar studies conducted with other software, such as Excel, with only minimal differences in outcomes [3]. Larger discrepancies observed in studies involving different materials highlight the importance of considering material properties in thermal simulations. MATLAB has proven to be a reliable and accurate tool for modeling heat distribution, contributing to improved thermal design in aerospace and other engineering applications.

The simulation results obtained using MATLAB show detailed temperature distributions across various grid sizes. The visualizations of heat distribution from the different grid divisions indicate that the highest temperature is located at the top edge of the plate, decreasing towards the bottom edge. This behavior aligns with the specified boundary conditions and the fundamental principles of heat transfer, where heat moves from regions of higher temperature to lower temperature. These results demonstrate that areas of the plate closer to the heat source (e.g., the aircraft engine) experience higher temperatures, while areas farther away exhibit lower temperatures.

This heat distribution pattern is particularly relevant for the thermal design of aircraft wings. Components of the wing that are near the engine must be designed to withstand high temperatures without compromising structural integrity or aerodynamic performance. Materials such as aluminum 7075, which was used in this research, show predictable and well-controlled temperature distributions, making them ideal for such applications. The application of these results can lead to more efficient and safer aircraft wing designs by ensuring even heat distribution and avoiding hotspots that could cause damage or structural failure.

4. Conclusions

From the calculations and simulations, it can be concluded that the heat distribution process on an Aluminum 7075 square plate, applying the Laplace equation using the 2023 version of the MATLAB Online application, achieved a numerical solution in the 9th iteration with an error of less than 1%, specifically 0.71%. When using MATLAB, the error at the 9th iteration was further reduced to 0.4681%. The heat distribution in equilibrium conditions on the aluminum plate is clearly and accurately represented using MATLAB, providing a better understanding of the temperature distribution pattern in this material. Comparisons with similar studies demonstrate consistency in the results, with only minimal differences attributed to variations in numerical precision and material properties. Factors that influence the appearance and accuracy of heat distribution include the grid size and the boundary conditions applied at the plate's edges. Based on the MATLAB simulation results using a 101-grid system, the heat distribution on the plate is more clearly visible, confirming that increasing the grid resolution improves both clarity and accuracy.

These findings provide valuable insights into heat distribution in aircraft wing materials, which is essential for efficient thermal design. A better understanding of temperature distribution patterns enables engineers to design aircraft components that can withstand heat while ensuring optimal performance. The use of materials like Aluminum 7075, which exhibits well-controlled temperature distribution, enhances the safety and operational efficiency of aircraft. This research also highlights the importance of simulation software, such as MATLAB, in conducting detailed thermal analyses and supporting the development of advanced aviation technologies. However, there are some limitations to this heat distribution analysis, particularly the use of a fixed grid size, which can affect the accuracy of the final results, and fixed boundary conditions, which may vary over time in real-world applications. Further research is recommended to explore simulations with higher resolution grids and dynamic boundary conditions to improve accuracy.

Supplementary Documentation

More detailed MATLAB coding has been provided in the Supplementary Document. To access the Supplementary Document, please visit the article homepage.

References

[1] J. M. Guerrero, A. Sasikumar, J. Llobet, and J. Costa, "Testing and simulation of a composite-aluminium wingbox subcomponent subjected to thermal loading," *Compos. Struct.*, vol. 296, p. 115887, 2022, doi: 10.1016/j.compstruct.2022.115887.

[2] B. Zhou, B. Liu, and S. Zhang, "The advancement of 7xxx series aluminum alloys for aircraft structures: A review," *Metals (Basel).*, vol. 11, no. 5, 2021, doi: 10.3390/met11050718.

[3] R. Samatowa, L., Kurniasari, S., Yunus, and M. Damopolii, "Penentuan distribusi suhu pada pelat logam dengan cara menyelesaikan persamaan diferensial eliptik menggunakan Microsoft Excel," *JPF (Jurnal Pendidik. Fis. Univ. Islam Negeri Alauddin Makassar)*, vol. 11, no. 1, p. 163, 2023.

[4] A. Noor and I. Fitrian, "Simulasi sebaran temperatur pelat logam tipis besi dan kuningan berbasis MATLAB," *Navig. Phys. J. Phys. Educ.*, vol. 2, no. 1, pp. 9–13, 2020.

[5] Y. Han and H. Dong, "Boundary Element Method for 3D Laplace and Stokes Flow Problems with Analytical Technique," *J. Phys. Conf. Ser.*, vol. 2650, no. 1, pp. 0–11, 2023, doi: 10.1088/1742-6596/2650/1/012030.

[6] K. Rachmawati and Vimala, "Simulasi perpindahan panas pada lapisan tengah pelat menggunakan metode elemen hingga," *J. Sains dan Seni ITS*, vol. 4, no. 2, pp. A13–A18, 2015, [Online]. Available: https://ejurnal.its.ac.id/index.php/sains_seni/article/view/10852%0Ahttps://ejurnal.its.ac.id

[7] S. C. Chapra and R. P. Canale, *Numerical methods for engineers*, 7th edition. New York: McGraw-Hill Education, 2015.

[8] E. B. D. M. Alves and J. R. Barbosa, "Laplace transform-based modeling of heat transfer in producing wellbores," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 45, no. 12, 2023, doi: 10.1007/s40430-023-04551-w.

[9] M. Kerr, G., González-Parra, and G. Sherman, "A new method based on the Laplace transform and Fourier series for solving linear neutral delay differential equations," *Appl. Math. Comput*. 420, 126914., 2022, doi: https://doi.org/10.1016/j.amc.2021.126914.

[10] B. Sudia, L. K. Mangalla, S. Samhuddin, and W. O. Zulkaidah, "Analisis Perpindahan Panas Pada Kolektor Plat Datar Sebagai Sumber Energi Termal Pada Pengering Tipe Rak," *Din. J. Ilm. Tek. Mesin*, vol. 10, no. 2, pp. 47–52, 2019, doi: 10.5281/zenodo.3036968.

[11] M. Fatwa, R. Rizki, P. Sriwinarty, and E. Supriyadi, "Pengaplikasian Matlab pada perhitungan matriks," *Papanda J. Math. Sci. Res.*, vol. 1, no. 2, pp. 81–93, 2022, doi: 10.56916/pjmsr.v1i2.260.

[12] M. P. Coleman, *An introduction to partial differential equations with MATLAB®*, Second edition. CRC Press, 2016.

[13] E. Georgantzia, M. Gkantou, and G. S. Kamaris, "Aluminium alloys as structural material: A review of research," *Eng. Struct.*, vol. 227, p. 111372, 2021, doi: 10.1016/j.engstruct.2020.111372.

[14] R. Padmanaban, V. Ratna Kishore, and V. Balusamy, "Numerical simulation of temperature distribution and material flow during friction stir welding of dissimilar aluminum alloys," *Procedia Eng.*, vol. 97, pp. 854–863, 2014, doi: 10.1016/j.proeng.2014.12.360.

[15] P. H. Shah and V. Badheka, "An experimental investigation of temperature distribution and joint properties of Al 7075 T651 friction stir welded aluminium alloys," *Procedia Technol.*, vol. 23, pp. 543–550, 2016, doi: 10.1016/j.protcy.2016.03.061.

[16] S. A. Tsirkas, "Numerical simulation of the laser welding process for the prediction of temperature distribution on welded aluminium aircraft components," *Opt. Laser Technol.,* vol. 100, pp. 45–56, 2018, doi: 10.1016/j.optlastec.2017.09.046.