

## **A MATHEMATICAL MODELING TO DETERMINE SUPPLIER CONTRACTS IN TRANSPORTATION PLANNING**

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### **Abstract**

*Transportation plays some important roles in supply chain management. It contributes significantly towards a business performance. Therefore, a company needs to plan its transportation system. A good transportation planning is able to lead in either efficiency or responsiveness. This paper addresses to build a mathematical model of determination of supplier contracts in a transportation problem. In this case, a company already selected a supplier partnership or, in other words, third-party logistics (3PLs) who provides trucks to be rented. The 3PLs also offers three options of contracts to the company, and the company should sign which contracts that can minimize rental costs and meet the company's demands. This model is a linear-programming model. Using a numerical example, the problem is solved with LINGO.*

**Keywords:** *Planning, Transportation, 3PLs, Contracts, Mathematical Modelling*

### **INTRODUCTION**

Supply chain is one of the core of the business processes. One of the Key Performance Indicators (KPI) of company performances can be assessed by looking at the supply chain management inside the company.

There are three phases in supply chain management that a company should do to increase its performance. They are supply chain design or strategy, planning, and operation (Chopra, 2007). Supply chain design is related to long-term decisions, which the time period is more than one year (Chopra, 2007). For instance, building some distribution centre or warehouses in some locations to close to customers and buying some new machines for production activities. Another one is supply chain planning. The time period is between a quarter and a year (Chopra, 2007). The last phase is supply chain operation. This phase is a short-term decision which focus on some operational execution to get the jobs done, for an example monitoring the number of successful deliveries and dropped deliveries (Mangan, 2012).

One of the phases explained above is supply chain planning. It is a medium-term decision, which the result determines some operational policies related to supply chain operation (Chopra, 2007). This decision might change after approximately one year adjusted with the future changes. There are some roles that contribute in supply chain planning, like inventory, pricing, and transportation. However, transportation plays a significant role in this area. Transportation takes into account the most expensive logistics costs and has a notable amount of the selling price for some products (de Moura, 2016). Transportation also helps both shippers and consignees in their operational business for supply chain efficiency, minimization of logistics cost, and customer satisfaction (Ke, 2015). For instance, a company should decide whether the products are going to distributed directly to its customers or are

going to go through to an intermediate point. If the product is a software, it can be downloaded by customers after buying it without any physical shipment (Chopra, 2007). When the company distribute the products to customers, a company also should determine the transportation system. The company must decide if it will buy its own vehicles or rent them. This situation takes into account to sourcing activities.

Sourcing activities is a process how a company can provide its resources needed. In other words, an activities how a company obtain the resources (Mangan, 2012). Sourcing is some activities which cover suppliers or partners selection, order activities, contracts negotiation, and procurement process (Li, 2014). In sourcing activities, there are five phases, such as suppliers or partners selection, supplier contracts planning, product design collaboration, procurement activities, and supplier performance assessment (Chopra, 2007). A company can do sourcing by full filling its needs itself, like making some materials or doing services inside the company. However, it usually do outsourcing with the Third Party Logistics (3PLs). 3PLs is a supplier partnership outside the company which provides some services, such as warehousing or trucking, and probably does all or part of a company's material management and distribution function (Simchi & Levi, 2007). However, recently, 3PLs broadens a range of its business. It does not only work on trucking or warehousing, but also involves some business of supply chain consultancy, IT services, and order management (Langley & Capgemini, 2014).

Outsourcing is very necessary activity that a company should think about. Outsourcing is able to inflate surplus with less risks (Chopra, 2007), to cut costs (Jacoby, 2009; Wanke, 2012) and company's asset investments together with to increase its asset productivity (Langley & Capgemini, 2014). Furthermore, a company can only focus on its core business instead of also working on logistics expertise and put it to 3PLs (Chopra, 2007; Qureshi et al, 2007; Wanke, 2012).

The aim of this research project has therefore been to try and establish the mathematical model of ground transportation planning, in this case, trucks, by selecting some contracts with different periods offering by a supplier partnership. The objective function of this model is, indeed, to minimize the cost.

This paper starts with a literature review in Section 2 and methodology in Section 3. Then it will deal with a problem in Section 4, a mathematical model in Section 5, the problem solution in Section 6, and a conclusion in the last section.

## **LITERATURE REVIEW**

There are some published studies related to sourcing activities with the third-party logistics (3PLs). Kilbourn et al. (2017), Byrne et al. (2013), Bianchini (2018), and Aguezzoul and Pires (2016) did research on the third party logistics selection. Kilbourn et al. (2017) gathered some data from 103 out of Top 500 companies in Africa which used 3PLs to run their business. Based on the result using a factor analysis method, there are three critical criteria to choose a right partner: service quality, information management and compliance, and collaboration. Byrne et al. (2013) developed a new partner selection methodology using a computer based simulation for Dell. Bianchini (2018) and Aguezzoul and Pires (2016) used a multi-criteria decision-making methodology (MCDM) to evaluate and select the third-party logistics.

Another research also investigate the third-party logistics, which is more specific to order activities with the 3PLs. Order activities here mean how their products are able to be shipped from an origin to a destination using 3PLs. Baglio et al. (2017) offered three physical distribution models adopted from pharmaceutical companies in Italy. These models are

highly recommended for Italian 3PLs to adjust their services with the distribution strategies of the Italian pharmaceutical companies. Kumar et al. (2006) also proposed a mathematical model for an allocation problem of fish market in India.

This paper is an initial step to solve a problem related to negotiating or signing contract or supplier contract planning with 3PLs, which is one of phases sourcing activities. This paper proposes a model that is, hopefully, able to contribute in the future research.

## METHODOLOGY

In this paper, a description of the problem is presented. After that, a mathematical model is built. The model is synthesized using linier programming method. To test the model, a numerical examples is used to check the result whether it represent the mathematical model already created.

## RESULT AND DISCUSSION

### The Problem

A company already selected a third-party logistics (3PLs) for its transportation planning. The next phase is to negotiate the contracts. After having a look at the data, the company requires a lot of trucks which will be rented from the 3PLs. In January, the company still has 175 trucks rented from a vendor and the contract will finish by the end of January. In fact, demands of trucks are more than 175 in January and the total demands from February 2015 to December 2016. Hence, the company must renew the contract. A table below provides the demand forecast:

**Table 1.** Demands of Trucks in 2015 – 2016

	<b>2015</b>	<b>2016</b>
January	425	430
February	410	420
March	390	430
April	430	415
May	425	465
June	360	390
July	460	390
August	435	395
September	395	380
October	400	400
November	475	430
Desember	410	440

There are three options available to extend the contracts presented in a table below:

**Table 2.** Three Options of Contracts

<b>Contract</b>	<b>Contract's Period</b>	<b>Rental Cost</b>
1	3-month contract	\$ 1600/ van
2	5-month contract	\$ 2150/ van
3	6-month contract	\$ 2500/ van

These contract are always begun at the beginning of the month, and in the end of the year 2016, the company must have no remaining. To minimize the total cost as well as to satisfy the demands, which contracts should the company sign?

**A mathematical model**

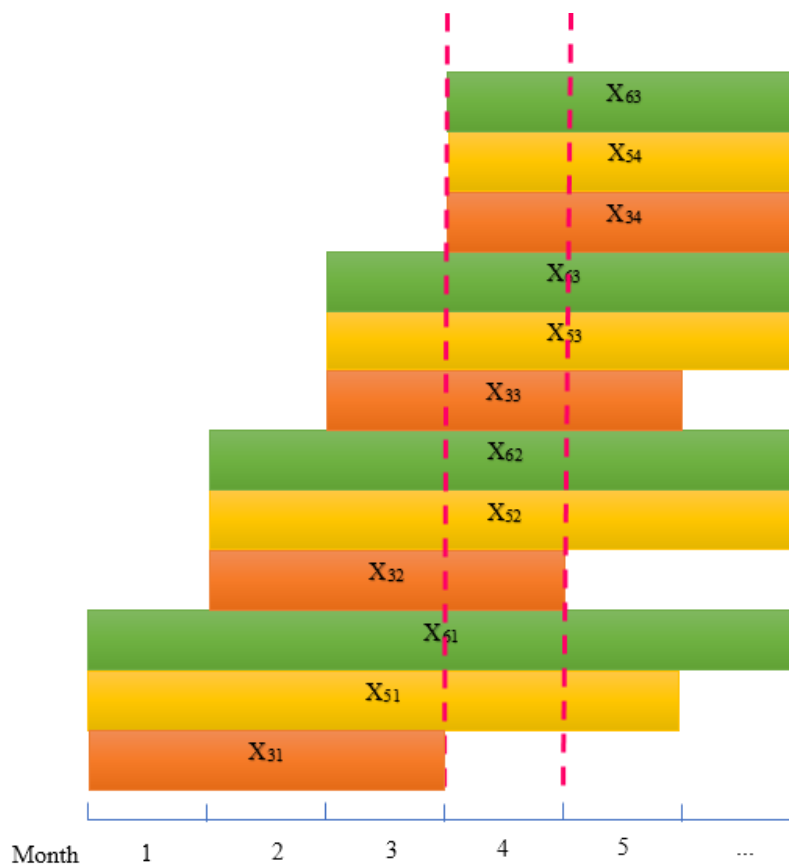
As an initial step, some notations based on the problem above should be written down into:

- $i$  : contracts.
- $j$  : months.
- $c_i$  : a cost of contract  $i$  to rent a truck.
- $x_{ij}$  : a number of trucks rented from a contract  $i$  for month  $j$ .
- $x_{im}$  : a number of trucks rented from a contract  $i$  for particular month  $j$ .
- $d_j$  : a demand forecast of trucks in month  $j$ .

Once the notations are already made, the next step is to determine a decision variable, constrains, and an objective function, which are:

- a decision variable: the number of trucks will be rented by signing a contract  $i$  in month  $j$  ( $x_{ij}$ ).
- Constrains : some contracts  $i$  are signed in some particular months  $j$  which should be satisfied a demand forecast of trucks in month  $j$  (constrain 1 and 2), and a number of trucks rented from a contract  $i$  for month  $j$  are integer (contract 3),  $n_j$  is the total number of months.
- An objective : to minimize total cost of trucks.

Prior to build the mathematical model, here is a brief explanation about contracts. For instance, contracts running in November 2016. Some possibility contracts existing in this month are 3-month contracts in September and October, 5-month contracts in July and August, as well as 6-month ones in June and July. However, remember, the company should find which contracts that can give optimum result. Figure 1 illustrates some contracts running in April 2015.



**Figure 1.** Some contracts running in April 2015

The next step is the general mathematical model, as follows:

Objective function:

$$\text{Minimize } \sum_{i \in \text{contracts}}^n \sum_{j \in \text{months}}^m c_i \times x_{ij} \quad (1)$$

Subject to:

$$\forall j = 1 : \text{initial} + \sum_{i \in \text{contracts}}^n \sum_{m=\max(1, j-i+1)}^{\min(j, nj-i+1)} x_{im} \geq d_j \quad (2)$$

$$\forall j=2, \dots, nj : \sum_{i \in \text{contracts}}^n \sum_{m=\max(1, j-i+1)}^{\min(j, nj-i+1)} x_{im} \geq d_j \quad (3)$$

$$\forall i \in \text{contracts}, j \in \text{months}: x_{ij} \text{ integer} \quad (4)$$

Taken together, the mathematical model of this problem will be:

Objective function:

$$\begin{aligned} \text{Minimize } & 1600x_{31} + 2150x_{51} + 2500x_{61} + 1600x_{32} + 2150x_{52} + 2500x_{62} + \\ & 1600x_{33} + 2150x_{53} + 2500x_{63} + 1600x_{34} + 2150x_{54} + 2500x_{64} + 1600x_{35} + \\ & 2150x_{55} + 2500x_{65} + 1600x_{36} + 2150x_{56} + 2500x_{66} + 1600x_{37} + 2150x_{57} + \\ & 2500x_{67} + 1600x_{38} + 2150x_{58} + 2500x_{68} + 1600x_{39} + 2150x_{59} + 2500x_{69} + \\ & 1600x_{310} + 2150x_{510} + 2500x_{610} + 1600x_{311} + 2150x_{511} + 2500x_{611} + \\ & 1600x_{312} + 2150x_{512} + 2500x_{612} + 1600x_{313} + 2150x_{513} + 2500x_{613} + \\ & 1600x_{314} + 2150x_{514} + 2500x_{614} + 1600x_{315} + 2150x_{515} + 2500x_{615} + \\ & 1600x_{316} + 2150x_{516} + 2500x_{616} + 1600x_{317} + 2150x_{517} + 2500x_{617} + \\ & 1600x_{318} + 2150x_{518} + 2500x_{618} + 1600x_{319} + 2150x_{519} + 2500x_{619} + \\ & 1600x_{320} + 2150x_{520} + 1600x_{321} + 1600x_{322} \end{aligned} \quad (5)$$

Constrains:

$$175 + x_{31} + x_{51} + x_{61} \geq 425 \quad (6)$$

$$x_{31} + x_{51} + x_{61} + x_{32} + x_{52} + x_{62} \geq 410 \quad (7)$$

$$x_{31} + x_{51} + x_{61} + x_{32} + x_{52} + x_{62} + x_{33} + x_{53} + x_{63} \geq 390 \quad (8)$$

$$x_{51} + x_{61} + x_{32} + x_{52} + x_{62} + x_{33} + x_{53} + x_{63} + x_{34} + x_{54} + x_{64} \geq 430 \quad (9)$$

$$x_{51} + x_{61} + x_{52} + x_{62} + x_{33} + x_{53} + x_{63} + x_{34} + x_{54} + x_{64} + x_{35} + x_{55} + x_{65} \geq 425 \quad (10)$$

$$x_{61} + x_{52} + x_{62} + x_{53} + x_{63} + x_{34} + x_{54} + x_{64} + x_{35} + x_{55} + x_{65} + x_{36} + x_{56} + x_{66} \geq 360 \quad (11)$$

$$x_{62} + x_{53} + x_{63} + x_{54} + x_{64} + x_{35} + x_{55} + x_{65} + x_{36} + x_{56} + x_{66} + x_{37} + x_{57} + x_{67} \geq 460 \quad (12)$$

$$x_{63} + x_{54} + x_{64} + x_{55} + x_{65} + x_{56} + x_{66} + x_{37} + x_{57} + x_{67} + x_{38} + x_{58} + x_{68} \geq 435 \quad (13)$$

$$x_{64} + x_{55} + x_{65} + x_{56} + x_{66} + x_{37} + x_{57} + x_{67} + x_{38} + x_{58} + x_{68} + x_{39} + x_{59} + x_{69} \geq 395 \quad (14)$$

$$x_{65} + x_{56} + x_{66} + x_{57} + x_{67} + x_{38} + x_{58} + x_{68} + x_{39} + x_{59} + x_{69} + x_{310} + x_{510} + x_{610} \geq 400 \quad (15)$$

$$x_{66} + x_{57} + x_{67} + x_{58} + x_{68} + x_{39} + x_{59} + x_{69} + x_{310} + x_{510} + x_{610} + x_{311} + x_{511} + x_{611} \geq 475 \quad (16)$$

$$x_{67} + x_{58} + x_{68} + x_{59} + x_{69} + x_{310} + x_{510} + x_{610} + x_{311} + x_{511} + x_{611} + x_{312} + x_{512} + x_{612} \geq 410 \quad (17)$$

$$x_{68} + x_{59} + x_{69} + x_{510} + x_{610} + x_{311} + x_{511} + x_{611} + x_{312} + x_{512} + x_{612} + x_{313} + x_{513} + x_{613} \geq 430 \quad (18)$$

$$x_{69} + x_{510} + x_{610} + x_{511} + x_{611} + x_{312} + x_{512} + x_{612} + x_{313} + x_{513} + x_{613} + x_{314} + x_{514} + x_{614} \geq 420 \quad (19)$$

$$x_{610} + x_{511} + x_{611} + x_{512} + x_{612} + x_{313} + x_{513} + x_{613} + x_{314} + x_{514} + x_{614} + x_{315} + x_{515} + x_{615} \geq 430 \quad (20)$$

$$x_{611} + x_{512} + x_{612} + x_{513} + x_{613} + x_{314} + x_{514} + x_{614} + x_{315} + x_{515} + x_{615} + x_{316} + x_{516} + x_{616} \geq 415 \quad (21)$$

$$x_{612} + x_{513} + x_{613} + x_{514} + x_{614} + x_{315} + x_{515} + x_{615} + x_{316} + x_{516} + x_{616} + x_{317} + x_{517} + x_{617} \geq 465 \quad (22)$$

$$x_{613} + x_{514} + x_{614} + x_{515} + x_{615} + x_{316} + x_{516} + x_{616} + x_{317} + x_{517} + x_{617} + x_{318} + x_{518} + x_{618} \geq 390 \quad (23)$$

$$x_{614} + x_{515} + x_{615} + x_{516} + x_{616} + x_{317} + x_{517} + x_{617} + x_{318} + x_{518} + x_{618} + x_{319} + x_{519} + x_{619} \geq 390 \quad (24)$$

$$x_{615} + x_{516} + x_{616} + x_{517} + x_{617} + x_{318} + x_{518} + x_{618} + x_{319} + x_{519} + x_{619} + x_{320} + x_{520} \geq 395 \quad (25)$$

$$x_{616} + x_{517} + x_{617} + x_{518} + x_{618} + x_{319} + x_{519} + x_{619} + x_{320} + x_{520} + x_{321} \geq 380 \quad (26)$$

$$x_{617} + x_{518} + x_{618} + x_{519} + x_{619} + x_{320} + x_{520} + x_{321} + x_{322} \geq 400 \quad (27)$$

$$x_{618} + x_{519} + x_{619} + x_{520} + x_{321} + x_{322} \geq 430 \quad (28)$$

$$x_{322} + x_{520} + x_{619} \geq 440 \quad (29)$$

### The problem solution

The problem is solved by LINGO. The total minimum cost is \$4.221.250 with 31 iterations. The result of the number of trucks are set out in Table 3.

**Table 3.** Number of Trucks Rented and Contracts Signed

Year	Month	Contract's Period			Total Trucks Rented
		3-month	5-month	6-month	
<b>2015</b>	January	15	65	170	250
	February	5	0	155	160
	March	0	0	0	0
	April	0	35	0	35
	May	0	0	0	0
	June	0	0	0	0
	July	0	140	130	270
	August	0	130	0	130
	September	0	0	0	0
	October	0	0	0	0
	November	45	30	0	75
	December	0	0	75	75
<b>2016</b>	January	0	0	280	280
	February	0	35	0	35
	March	0	10	0	10
	April	0	15	0	15
	May	0	50	0	50
	June	0	0	0	0
	July	0	0	315	315
	August	0	15	0	15
	September	0	0	0	0
	October	110	0	0	110
	November	0	0	0	0
	December	0	0	0	0
Total trucks rented in two years					<b>1825</b>

Table 3 presents the number of trucks rented in every month from January 2015 to December 2016 by signing some particular type of contracts. The company rents 1825 trucks during two years. The numbers in the table is a number of trucks rented in every type of contracts (a 3-month contract, a 5-month contract, and a 6-month contract), if any, and total number of trucks rented in every month.

Three 3-month contracts are signed in January and November 2015 also October 2016. Eight 5-month contracts are signed in January, July, August, and November 2015 as well as from February to May along with August 2016. Six 6-month contracts are signed in January, February, July, and December 2015 as well as January and July 2016. In addition, in 2015, there are 250 trucks rented in January, 160 trucks in February, 270 trucks in July, 130 trucks in August, 75 trucks in November also December. In 2016, there are 280 trucks rented in January, 35 trucks in February, 10 trucks in March, 15 trucks in April, 50 trucks in May, 315 trucks in July, 15 trucks in August, and 110 trucks in October.

Are the trucks satisfied the demand and is there no remaining trucks in the end of 2016? Table 4 is shown how the model satisfy the demands without ignoring the requirement that the company must have no remaining trucks in the end of the year 2016.

**Table 4.** Number of Demands Satisfied By The Model

Year	Month	A Number of Remaining Trucks	A Number of Trucks Rented	Total Number of Trucks Available	Total Number of Trucks Required
2015	January	175	250	425	425
	February	250	160	410	410
	March	410	0	410	390
	April	395	35	430	430
	May	430	0	430	425
	June	365	0	365	360
	July	190	270	460	460
	August	305	130	435	435
	September	400	0	400	395
	October	400	0	400	400
	November	400	75	475	475
	December	335	75	410	410
2016	January	150	280	430	430
	February	430	35	465	420
	March	465	10	475	430
	April	400	15	415	415
	May	415	50	465	465
	June	390	0	390	390
	July	75	315	390	390
	August	380	15	395	395
	September	380	0	380	380
	October	330	110	440	400
	November	440	0	440	430
	December	440	0	440	440

Table 4 provides how the contracts or the number of trucks rented can meet the demand in every month. Initially it is given in the problem that the company has 175 trucks in the beginning of January and the contract will be finished in the end of January. In addition, 250 trucks are rented in January. Thus, the total number of trucks is 425 trucks satisfying the demand. In February, the number of remaining trucks are 250 trucks, and the company rents 160 trucks. Hence, the total number of trucks is 410 trucks also satisfying the demand. In March, the company does not rent any trucks. However, the contracts running in that month are two 3-month contracts, one 5-month contract, and two 6-month contracts, which the total number of trucks is 410 trucks. Those trucks can full fill the demand of 390 trucks. In April, a 3-month contract in January is expired so that the company lose 15 trucks out of 410 trucks. However, 35 trucks are rented. Hence, the company still has 430 trucks and can meet the demand. These calculations were the same for the rest of the months until December 2016 considering the total number of remaining trucks from the previous contracts signed before and the number of trucks rented in the particular month. In summary, these results indicate that all demands throughout the year can be complied with the total number of trucks rented.

**CONCLUSION**

From the numerical example, the mathematical model can solve the problem. The company is able to decide which contracts that should be signed and how many trucks that should be rent to meet the demands in two years. However, further work needs to be done to establish more complex problem with, indeed, more complicated mathematical model.



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